Cams, Gears, and Splines

Learning Objectives

After studying this chapter, you will be able to:

- Explain the purpose of cams.
- Identify and describe the basic types of cams.
- Describe the different types of cam motion.
- Explain how spur gears, bevel gears, and worm gears function.
- List and describe common terms related to gears, bevel gears, and worm gears.
- Explain the purpose of splines.
- Make working drawings of cams and gears.

Technical Terms

Addendum
Addendum angle
Axial pitch
Backing
Base circle
Bevel gears
Cam
Cam follower motion
Center distance
Chordal addendum
Chordal thickness
Circular pitch
Circular thickness
Clearance
Combination motion
Cone distance
Crown backing
Crown height
Cylindrical cam
Dedendum
Dedendum angle
Diametral pitch
Displacement
Displacement diagram
Dwell
Face angle
Face length
Face radius
Face width
Follower
Gears
Groove cam
Harmonic motion
Lead
Lead angle
Line of action
Miter gears
Mounting distance
Number of teeth
Outside diameter
Pinion
Pitch angle
Pitch apex
Pitch circle
Pitch cone
Pitch diameter
Plate cam
Pressure angle
Rack
Root angle
Root diameter
Shaft angle
Splines
Spur gears
Straight spur gears
Throat diameter
Uniform motion
Uniformly accelerated motion
Whole depth
Working depth
Worm gears
Worm mesh

Many types of machines require mechanisms to transfer motion and power from one source to another. Most often this transfer has to occur without slippage that might be present with belts. It is also necessary in some instances to convert rotary motion to reciprocal motion (straight-line motion) at a certain rate of speed for related parts. For example, the firing action of a four-stroke internal combustion engine converts reciprocal motion to circular motion, **Figure 19-1**.

In four-stroke engine operation, there must be a specific timing for opening and closing of the valves in relation to the cycling of the piston. This is achieved with gears and cams. This chapter discusses basic types of cams, gears, and splines, and how they are represented on drawings.

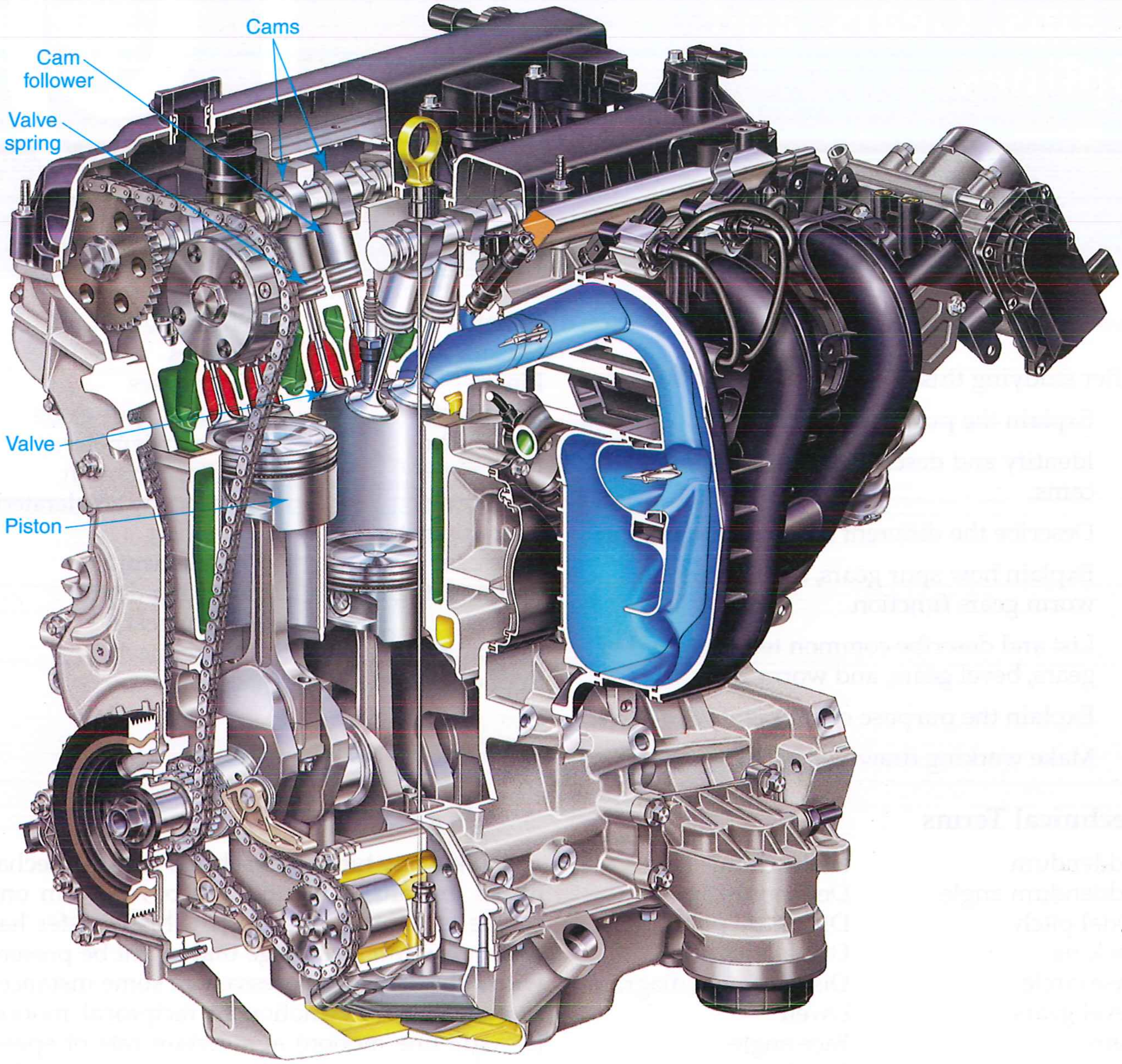

Figure 19-1. Cams are used to maintain the precise relationship between valves and pistons to produce the intake, compression, power, and exhaust strokes of an internal combustion engine. Shown is a four-cylinder, dual overhead cam engine. (Ford)

Cams

A ***cam*** is a mechanical device that changes uniform rotating motion into reciprocating motion of varying speed. Three types of cams are commonly used. These are the ***plate cam***, ***groove cam***, and ***cylindrical cam***. Different types of plate, groove, and cylindrical cams are shown in **Figure 19-2**.

A ***follower*** makes contact with the surface or groove of the cam. The follower is held against the cam by gravity, spring action, or by a groove in the groove cam. The basic types of cam followers are the *knife edge follower, flat face follower*, and *roller follower*, **Figure 19-3**. These followers pick up the rotating motion of the cam and change it to reciprocating motion.

Figure 19-2. Common types of cams commonly used in mechanisms. A—Plate cams. B—Groove cams. C—Cylindrical (or barrel-shaped) cams. (Industrial Motion Control, LLC/Camco-Ferguson)

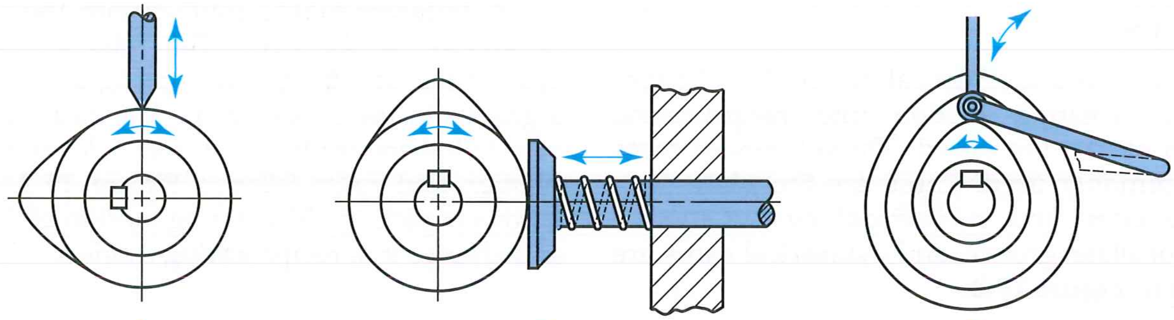

Figure 19-3. The three basic types of cam followers. A—Knife edge. B—Flat face. C—Roller.

Cams may be designed to provide a number of different types of motion and displacement patterns. ***Cam follower motion*** refers to the cam follower's rate of speed or movement in relation to the uniform rotation speed of the cam, **Figure 19-4**. ***Displacement*** refers to the distance the cam follower moves in relation to the rotation of the cam.

Cam Displacement Diagrams

A ***displacement diagram*** is a graph or drawing of the displacement (travel) pattern of the cam follower caused by one rotation of the cam, **Figure 19-5**. Construction of a displacement diagram is usually the first step in the design of a cam.

Referring to **Figure 19-5**, the length of the baseline represents one revolution of the cam through 360°. The divisions on the baseline, or angular sectors, represent time intervals of the revolving cam. When the speed of rotation of a cam is constant, the time intervals are uniform. When the time intervals are kept constant, the cam follower's rate of speed varies as the angle or incline of the cam changes.

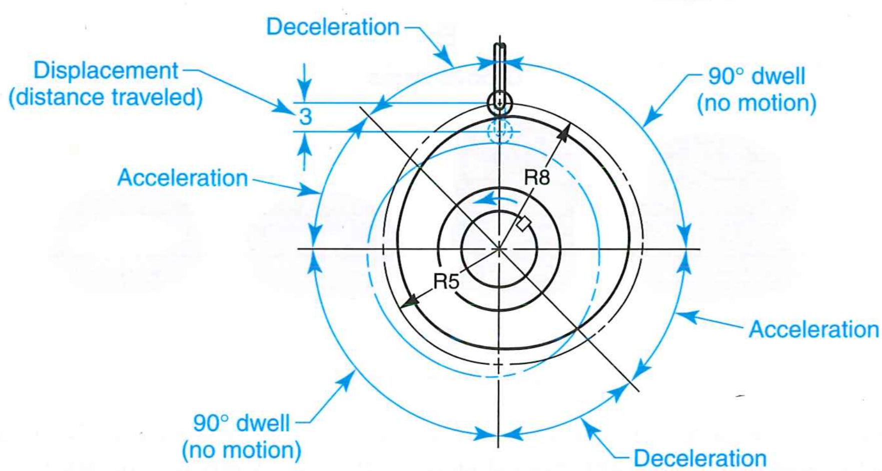

Figure 19-4. Cams can be designed to provide a variety of motion and displacement patterns.

The height of the diagram measured from the baseline represents the distance of travel or displacement of the cam follower. The shape of the curve determines the type of cam follower motion. (The common types of cam follower motion are discussed in the next section.) Intersection points on the curve in the diagram represent displacement measurements (or *ordinates*) that are used in the cam layout. Since the displacement diagram is only representative of the motion of the cam, the divisions along the baseline are approximations of the actual spaces on the cam base circle in the cam layout. It is important to note that the displacement ordinates must be accurate since they are used in laying off measurements on the cam layout itself.

Types of Cam Follower Motion

The three principal types of motion for cam followers are uniform motion, harmonic motion, and uniformly accelerated motion. The different types of cam follower motion and cam layout procedures for each type are discussed in the following sections. Procedures for both manual and CAD drafting are presented.

Uniform motion

Uniform motion is produced when the follower moves at the same rate of speed from the beginning to the end of the displacement cycle. The shape of a uniform motion cam is shown as a straight line in the displacement diagram, **Figure 19-6A**.

With a uniform motion cam design, the starting and stopping of the follower is very abrupt due to instantaneous changes in velocity. So the cam shape is usually modified with arcs having a radius of one-fourth to one-half the follower displacement. This smoothens out the beginning and ending of the follower stroke. **Figure 19-6A** shows a displacement layout for both a uniform motion cam and a modified uniform motion cam. The uniform motion cam is used for machinery operating at a slow rate of speed.

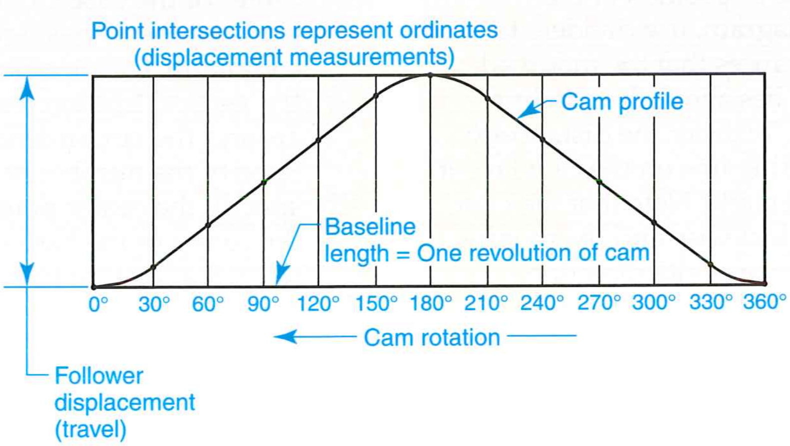

Figure 19-5. A cam displacement diagram is a graph of the travel pattern of the cam follower caused by one rotation of the cam. Creating the diagram is usually the first step in designing a cam.

Construct a Modified Uniform Motion Cam

Using Instruments (Manual Procedure)

If you are drawing manually, proceed as follows to lay out a modified uniform motion cam from the displacement diagram shown in **Figure 19-6A**. A CAD procedure for this construction is presented after this procedure.

1. Lay out a base circle of a specified radius. Refer to **Figure 19-6B**. The radius is equal to the distance from the cam axis to the lowest follower position (as shown at the 0° position).
2. Draw a convenient number of equally spaced radial lines dividing the base circle into intervals representing the angular motion of the cam. (The number of divisions must equal the divisions along the baseline of the displacement diagram for the cam.) In **Figure 19-6**, 24 increments of 15° have been used.
3. Starting with the 0° position of the displacement diagram, use dividers to transfer the distances that the modified cam profile line lies above the baseline on each 15° line. Transfer the distances to each corresponding line on the cam layout beyond the base circle. Note that the cam rotates counterclockwise and the plotting progresses in the opposite direction.
4. When all points have been located, sketch a smooth curve through the points. Finish with an irregular curve.

Using the Line and Spline Commands (CAD Procedure)

The same drawing principles used in laying out cam profiles in manual drafting apply to CAD drafting. However, CAD drawing commands help simplify the process. The following procedure uses the **Line** and **Spline** commands to construct a cam profile by laying out measurements from a displacement diagram. In more advanced CAD programs, special tools may be available to create cam profiles based on the specific engineering data, such as the type of cam motion, body diameter, groove depth, and radial dimensions defining rise and dwell. To construct a cam profile from a displacement diagram, use the following procedure. Refer to **Figure 19-6**.

1. Enter the **Circle** command. Draw a base circle of a specified radius. Refer to **Figure 19-6B**.
2. Enter the **Line** command. Using object snaps, draw a vertical line from the center of the base circle to the top of the circle (at the 0° position). Enter the **Array** command. Create a polar array and array the vertical line to create the radial lines around the circumference of the circle. Specify the number of items as 24 and specify the center point of the array as the center of the base circle. This creates the radial lines at increments of 15°.

3. Using direct distance entry and polar tracking with 15° increments, draw a series of lines from the intersection points on the base circle to locate the points on the profile curve. Use the **Distance Between Two Points** calculator function to calculate the ordinate distances from the displacement diagram. Use direct distance entry to draw the lines at the calculated distances and the appropriate polar angles. Work in a clockwise direction.
4. Enter the **Spline** command. Using object snaps, draw a spline through the located points.

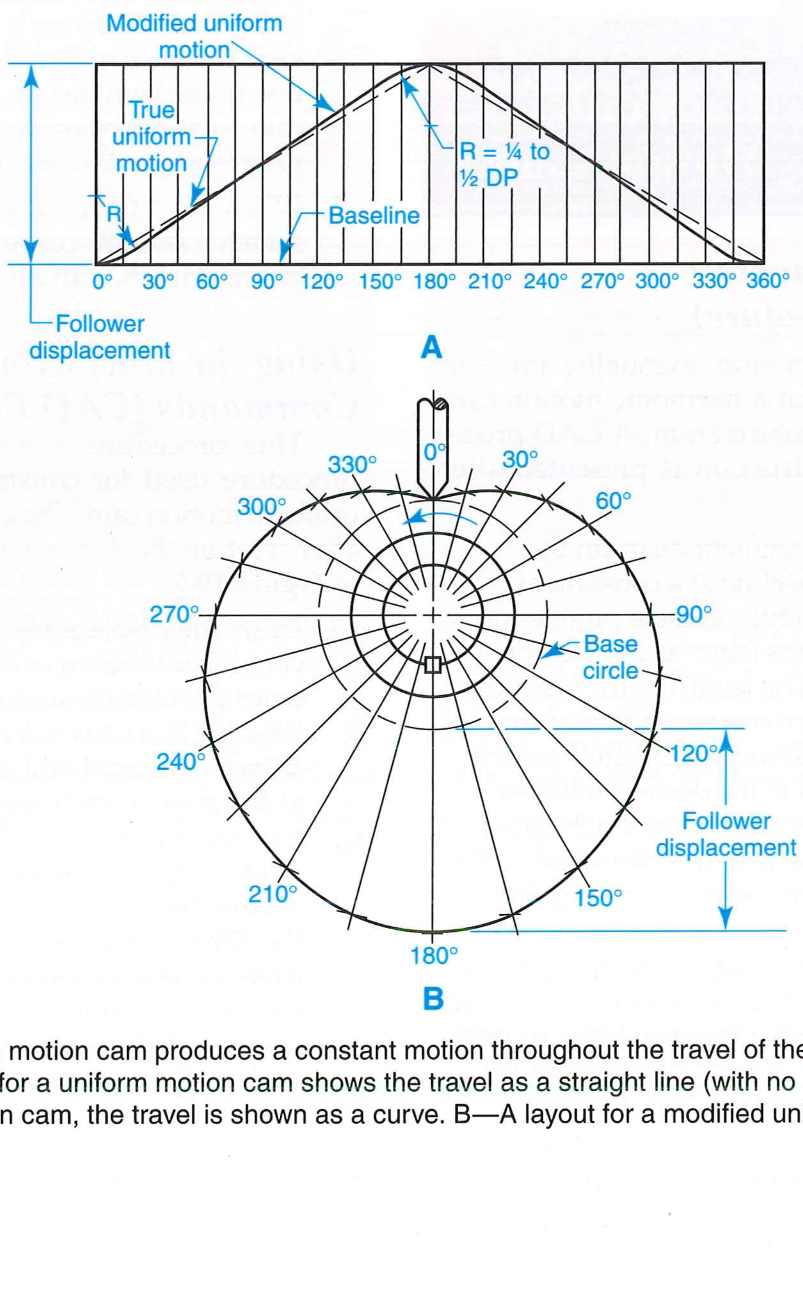

Figure 19-6. A uniform motion cam produces a constant motion throughout the travel of the follower. A—The displacement diagram for a uniform motion cam shows the travel as a straight line (with no curves). For a modified uniform motion cam, the travel is shown as a curve. B—A layout for a modified uniform motion cam.

Harmonic motion

Harmonic motion is produced when the movement of the cam follower is a smooth, continuous motion. This movement is based on the successive positions of a point moving at a constant velocity around the circumference of a circle, **Figure 19-7**. The harmonic motion cam is used for machinery operating at moderate speeds.

CAD

Construct a Harmonic Motion Cam

Using Instruments (Manual Procedure)

If you are drawing manually, proceed as follows to lay out a harmonic motion cam from a displacement diagram. A CAD procedure for this construction is presented after this procedure.

1. Lay out a displacement diagram by drawing the baseline at a convenient length and dividing it into a number of angular divisions (such as 24). Refer to **Figure 19-7A**. The length of the diagram should represent one revolution of the cam. Next, construct a semicircle with a diameter equal to the desired follower displacement at one end of the diagram. Divide the semicircle into the same number of equal parts as there are angular divisions for one-half of the cam layout. Project these divisions horizontally to the corresponding angular divisions to locate the displacement ordinates. Draw a curve through the displacement ordinates to complete the displacement diagram.
2. Lay out a base circle of a specified radius. Refer to **Figure 19-7B**. The radius is equal to the distance from the cam axis to the lowest follower position (as shown at the 0° position).
3. Draw a convenient number of equally spaced radial lines dividing the base circle into sectors representing the angular motion of the cam. (The number of divisions must equal the divisions along the baseline of the displacement diagram.)
4. Starting with the 0° position of the displacement diagram, transfer the distances that the harmonic curve lies off of the baseline at each ordinate. Transfer each distance to its corresponding radial line in the cam layout. Note that the cam rotates clockwise and the plotting progresses in the opposite direction.
5. When all of the points have been located, sketch a smooth curve through the points. Finish with an irregular curve.

Using the Line, Xline, and Spline Commands (CAD Procedure)

This procedure is similar to the CAD procedure used for constructing a modified uniform motion cam. The cam profile is drawn after creating the displacement diagram. Refer to **Figure 19-7**.

1. Draw the displacement diagram for the cam layout shown in **Figure 19-7A**. First, enter the **Line** command and draw the baseline at a convenient length. Enter the **Offset** command and offset the baseline at the displacement distance to create the upper horizontal line of the diagram. Then, enter the **Divide** command and divide the baseline into 24 parts. Enter the **Xline** command. Using object snaps, draw vertical construction lines through the division points. Next, enter the **Circle** command and use the **Two Points** option to draw a circle at the end of the diagram. Use object snaps to select the endpoints of the baseline and the offset line. Enter the **Divide** command and divide the circle into 24 parts. Enter the **Xline**

command. From the division points on the left half of the circle, draw horizontal construction lines to intersect the vertical construction lines in the diagram. The intersections of the construction lines locate the displacement ordinates. Enter the **Spline** command. Draw a spline through the located points to complete the displacement diagram.

2. Enter the **Circle** command. Draw a base circle of a specified radius. Refer to **Figure 19-7B**.
3. Enter the **Line** command. Using object snaps, draw a vertical line from the center of the base circle to the top of the circle (at the 0° position). Enter the **Array** command. Create a polar array and array the vertical line to create the radial lines around the circumference of the circle.
4. Using direct distance entry and polar tracking, draw a series of lines from the intersection points on the base circle to locate the points on the profile curve. Use the **Distance Between Two Points** calculator function to calculate the ordinate distances from the displacement diagram. Use direct distance entry to draw the lines at the calculated distances and the appropriate polar angles. Work in a counterclockwise direction.
5. Enter the **Spline** command. Using object snaps, draw a spline through the located points.

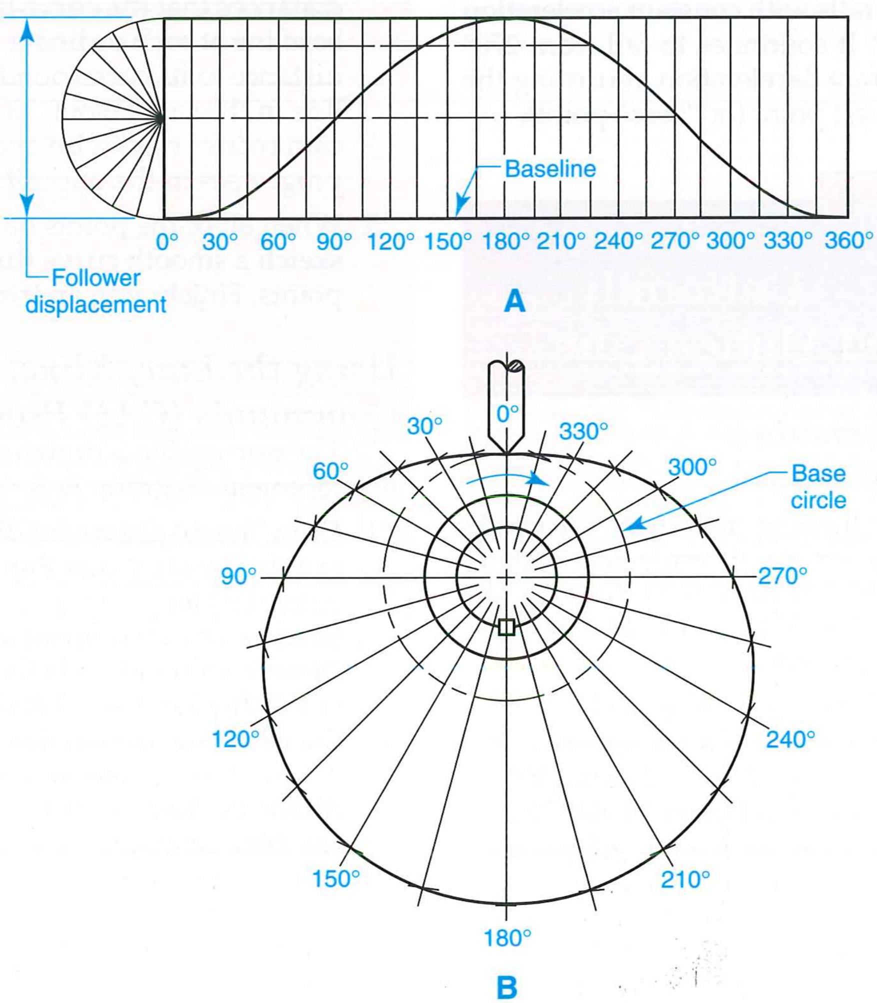

Figure 19-7. A harmonic motion cam moves the follower at a constant speed. A—The displacement diagram shows the travel as a smooth curve. B—The cam layout.

Uniformly accelerated motion

Uniformly accelerated motion is designed in a cam to provide constant acceleration or deceleration of the follower displacement. See **Figure 19-8**. This type of motion is suited for high-speed cam operation.

On the displacement diagram, the divisions along the baseline represent equal intervals of time. The displacement ordinates vary so that they are proportional to the squares of successive time intervals on the baseline (such as $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, and so on).

During the first half-revolution of the cam, the follower rises with constant acceleration from 0° to 90°. From 90° to 180°, the cam still rises, but with constant deceleration. Note on the displacement diagram that the ordinate scale reverses at 90° (or midway). Refer to **Figure 19-8A**.

During the second half-revolution of the cam, the follower falls with constant acceleration from 180° to 270°. It continues to fall from 270° to 360° with constant deceleration, returning the follower to its lowest point (or "zero" point).

CAD Construct a Uniformly Accelerated Motion Cam

Using Instruments (Manual Procedure)

If you are drawing manually, proceed as follows to lay out a uniformly accelerated motion cam from a displacement diagram. A CAD procedure for this construction is presented after this procedure.

1. Lay out a displacement diagram by drawing the baseline at a convenient length and dividing it into 12 parts (30° increments). Refer to **Figure 19-8A**. The length of the diagram should represent one revolution of the cam. Lay out a displacement diagram by drawing an inclined line at a convenient angle and laying off squares of successive intervals of time. Refer to **Figure 19-8A**. Note that the squares of the intervals increase through the third interval (90°) and decrease in the same manner from the third interval to the height of the full displacement (180°). Project these divisions to the corresponding angular lines beginning at 0°. Draw a curve through the projected ordinates to complete the displacement diagram.
2. Lay out a base circle of a specified radius. Refer to **Figure 19-8B**. The radius is equal to the distance from the cam axis to the lowest follower position (as shown at the 0° position).
3. Draw 12 equally spaced radial lines dividing the base circle into sectors representing the angular motion of the cam.
4. Starting with the 0° position of the displacement diagram, transfer the distances that the curve lies off of the baseline at each ordinate. Transfer each distance to its corresponding radial line in the cam layout. Note that the cam rotates clockwise and the plotting progresses in the opposite direction.
5. When all of the points have been located, sketch a smooth curve through the points. Finish with an irregular curve.

Using the Line, Xline, and Spline Commands (CAD Procedure)

The cam profile is drawn after creating the displacement diagram. Refer to **Figure 19-8**.

1. Draw the displacement diagram for the cam layout shown in **Figure 19-8A**. First, enter the **Line** command and draw the baseline at a convenient length. Enter the **Offset** command and offset the baseline at the displacement distance to create the upper horizontal line of the diagram. Then, enter the **Divide** command and divide the baseline into 12 parts. Enter the **Xline** command. Using object snaps, draw vertical construction lines through the division points. Next, enter the **Line** command and use object snaps and tracking to draw the ordinate scale at the end of the diagram. Select the endpoint of the baseline as the first point of the

line and use object snap tracking to align the second point of the line with the endpoint of the offset line. Enter the **Divide** command and divide the line into 18 parts. Enter the **Xline** command. From the first, fourth, ninth, 14th, and 17th division points on the ordinate scale, draw horizontal construction lines to intersect the vertical construction lines in the diagram. The intersections of the construction lines locate the displacement ordinates. Enter the **Spline** command. Draw a spline through the located points to complete the displacement diagram.

2. Enter the **Circle** command. Draw a base circle of a specified radius. Refer to **Figure 19-8B**.
3. Enter the **Line** command. Using object snaps, draw a vertical line from the center of the base circle to the top of the circle (at the 0° position). Enter the **Array** command. Create a polar array and array the vertical line to create the radial lines around the circumference of the circle.
4. Using direct distance entry and polar tracking, draw a series of lines from the intersection points on the base circle to locate the points on the profile curve. Use the **Distance Between Two Points** calculator function to calculate the ordinate distances from the displacement diagram. Use direct distance entry to draw the lines at the calculated distances and the appropriate polar angles. Work in a counterclockwise direction.
5. Enter the **Spline** command. Using object snaps, draw a spline through the located points.

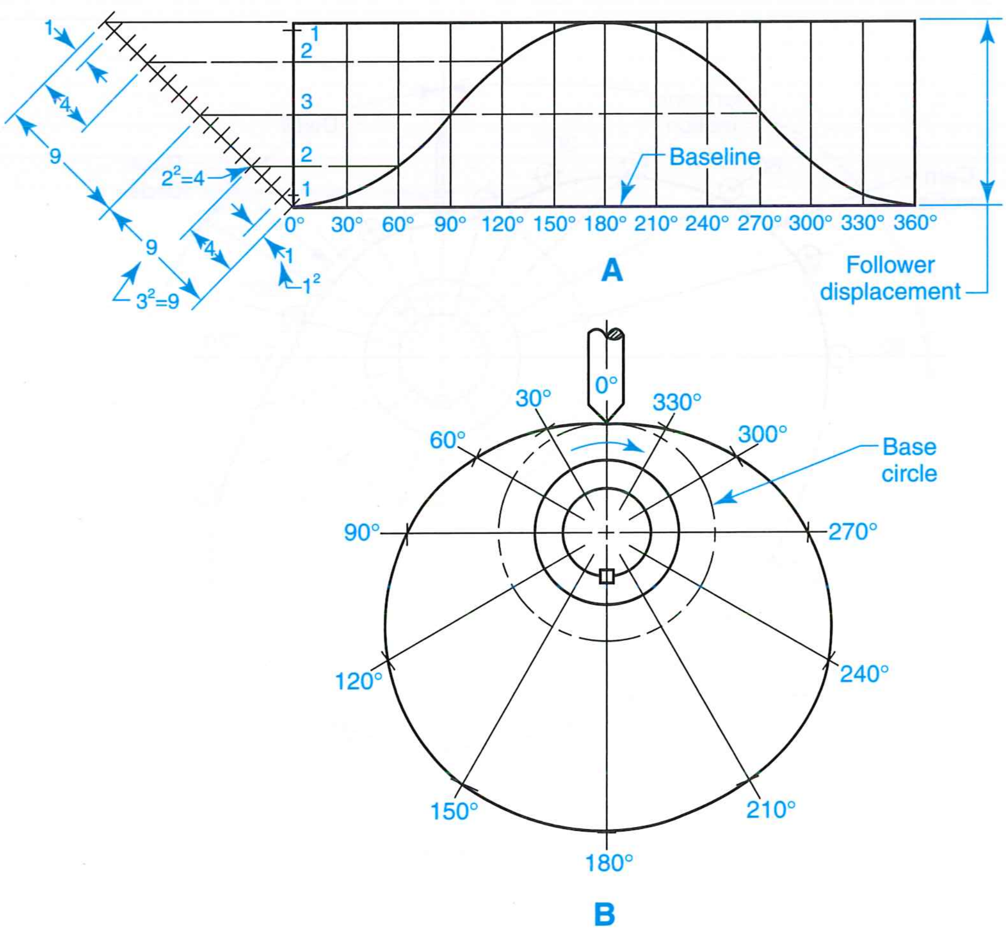

Figure 19-8. A uniformly accelerated motion cam provides uniform acceleration or deceleration of the follower. A—The displacement diagram shows the travel as a smooth curve. B—The cam layout.

Combination motion

Combination motion may be designed for a single cam in order to achieve the follower displacement desired, **Figure 19-9**. In the example shown, the cam design calls for a 90° period of harmonic motion, followed by a 90° period of dwell, a 120° period of uniform motion, and a 60° period of dwell. ***Dwell*** represents a period when the displacement remains unchanged. It is indicated on the displacement diagram as a horizontal line.

Note in **Figure 19-9B** that the cam follower is the roller type, and the center of the roller is assumed to start on the base circle for layout purposes. To construct the cam layout, transfer

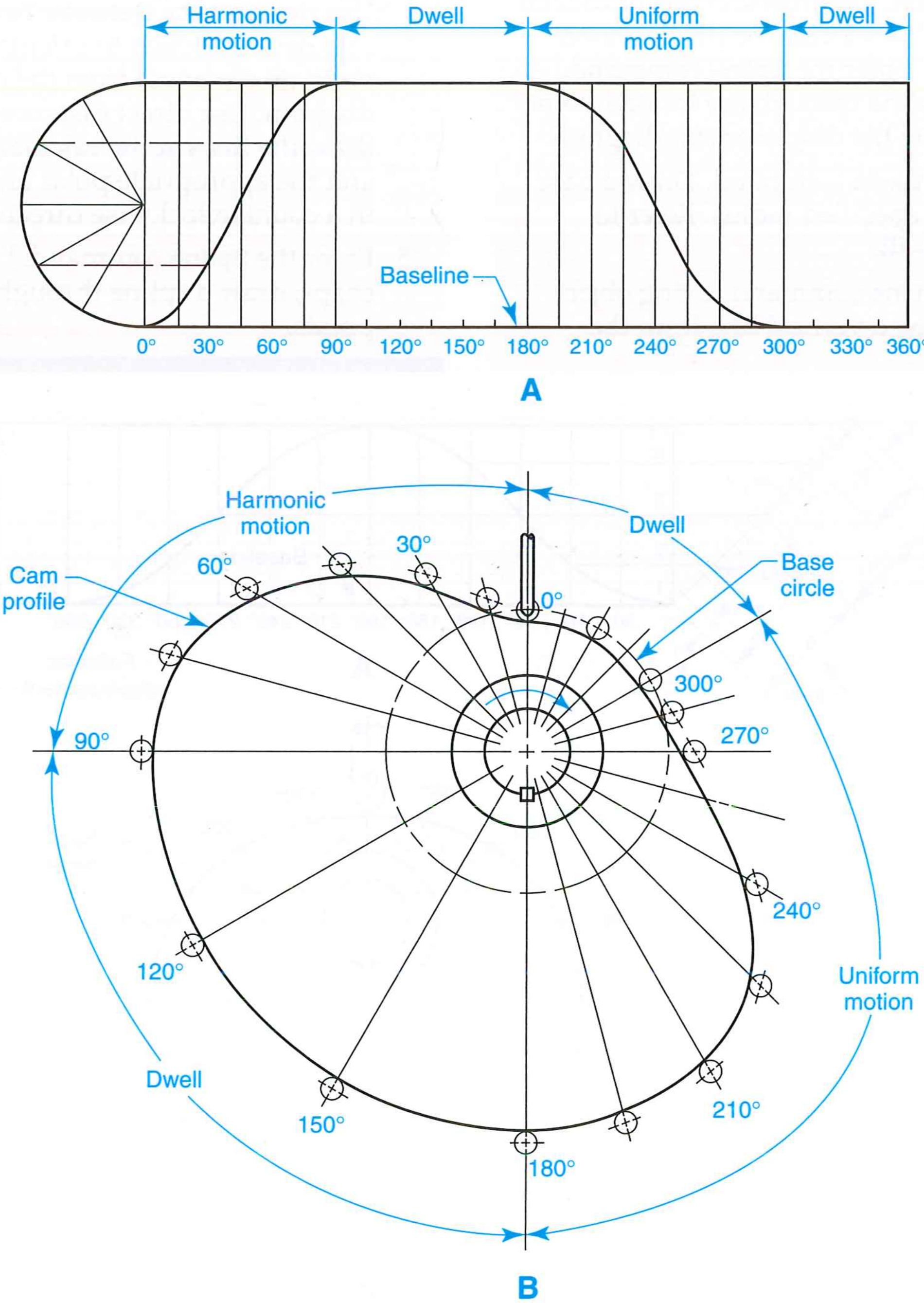

Figure 19-9. A combination motion cam incorporates different types of cam follower motion in its design. A—A cam design combining the characteristics of harmonic and uniform motion. B—The cam layout.

displacement distances from the diagram to the corresponding radial lines in the layout in the usual manner. Lay off an arc with a radius equal to that of the roller at each projected point in the layout. The cam profile is drawn tangent to the roller positions on the radial lines.

Designing a Cam with an Offset Roller Follower

A uniformly accelerated cam with an offset roller follower is shown in **Figure 19-10**. An offset roller follower has its centerline axis *offset* from the centerline axis of the cam. Note in **Figure 19-10** that the center of the roller follower is located on the base circle. Since the motion is uniformly accelerated throughout, the displacement distances are plotted directly from the follower using a special diagram.

To construct the cam layout, draw a circle with its center at the center of the base circle and tangent to the extended centerline of the roller follower. Divide this circle into 12 sections (30° increments) and draw tangents at the section points. Next, transfer distances from the diagram to the tangent lines from the base circle outward (as shown at the 90° radial line). Then draw circles representing the roller at each of these locations. Draw a smooth curve tangent to the 12 positions of the roller to form the cam profile.

Gears

Gears are machine parts used to transmit motion and power by means of successively engaging teeth. Gear teeth are shaped so contact between the teeth of mating gears is

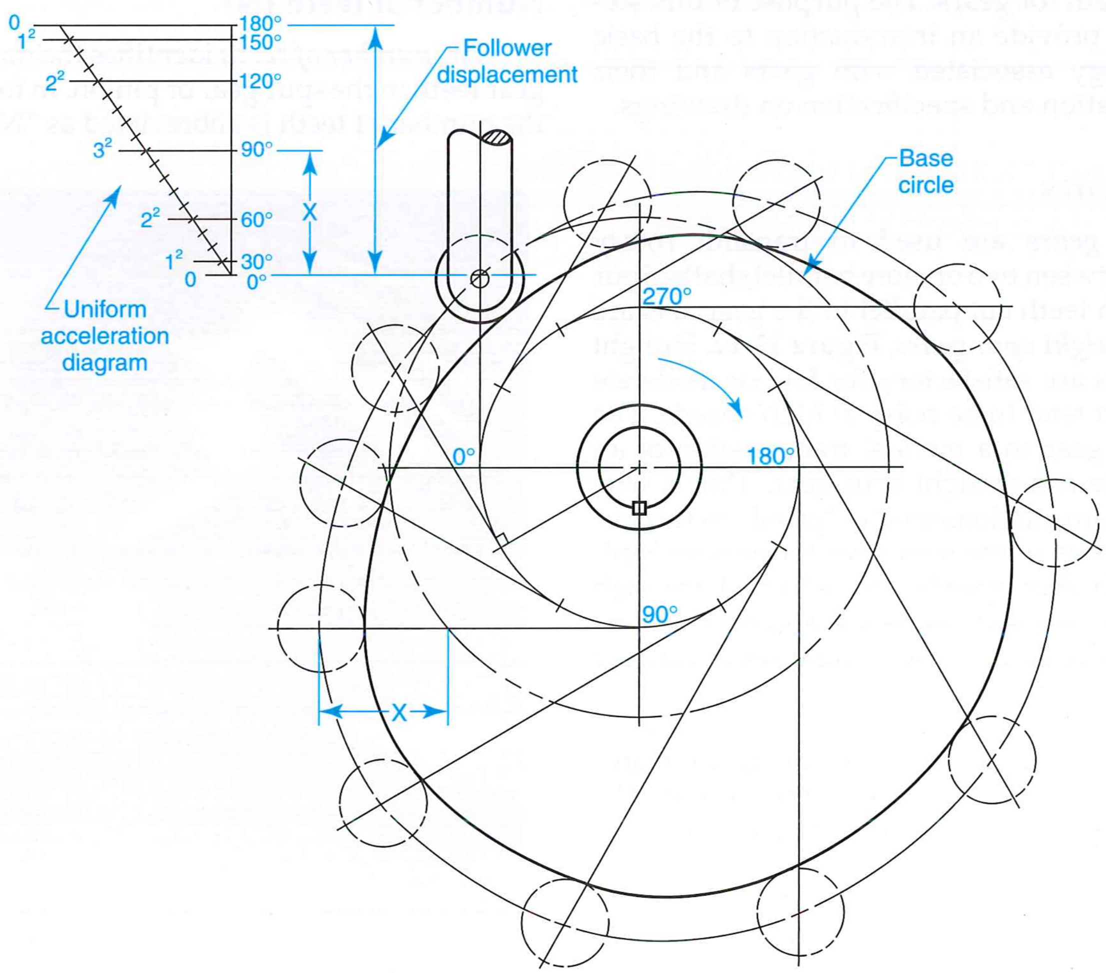

Figure 19-10. An offset roller follower has a centerline axis "offset" from the centerline axis of the cam. Shown is a cam design with uniformly accelerated motion.

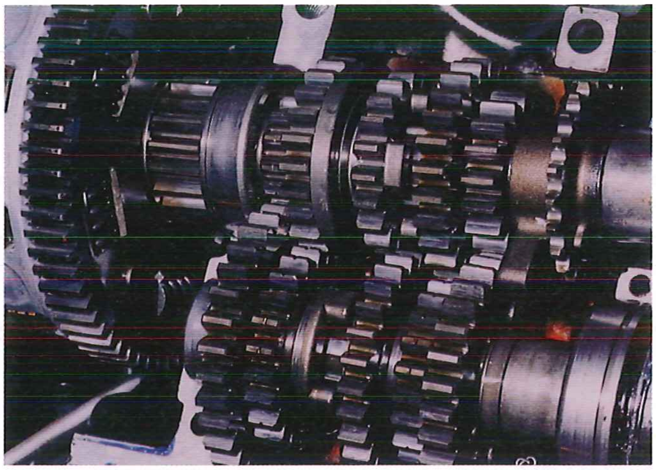

Figure 19-11. Gear assemblies are commonly used by machinery to transmit motion.

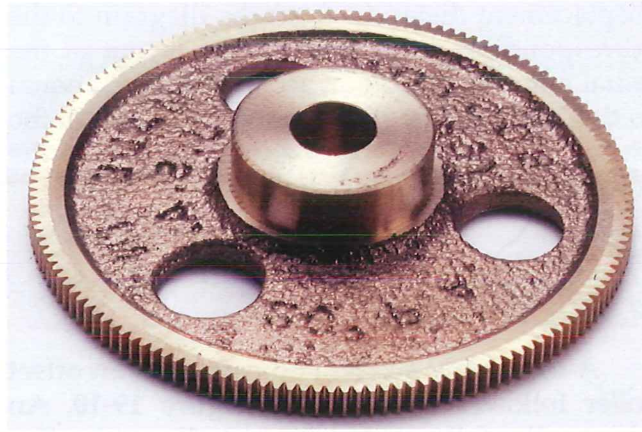

Figure 19-12. Straight spur gears have teeth cut parallel to the gear axis and are used to transmit motion between parallel shafts. (Boston Gear Co.)

continually maintained while rotation is occurring, **Figure 19-11**. Gear teeth designed from the involute curve are the most commonly used type of teeth for gears. The purpose of this section is to provide an introduction to the basic terminology associated with gears and their representation and specification on drawings.

Spur Gears

Spur gears are used to transmit rotary motion between two or more parallel shafts. Spur gears with teeth cut parallel to the gear axis are called ***straight spur gears***, **Figure 19-12**. Straight spur gears are satisfactory for low or moderate speeds but tend to be noisy at high speeds. The "reverse" gear in a manual transmission of an automobile is a straight spur gear. This is why manual transmissions tend to "grind" in reverse. Modifications of the spur gear for heavier loading and higher speeds are achieved through special designs such as helical and herringbone gears, **Figure 19-13**. Only straight spur gears are discussed in this chapter.

When mating spur gears of different size are in mesh, the larger one is called the gear (or spur gear), and the smaller one is the ***pinion***. The pinion is generally the drive gear, and the spur gear is generally the driven gear.

The drafter must know and understand terminology associated with gears in order to properly specify and represent gears on drawings. Some essential terms for spur gears are defined in the following sections and shown in **Figure 19-14**. Formulas are given, where appropriate, for determining various gear measurements.

Number of teeth (N)

The ***number of teeth*** identifies the number of gear teeth in the spur gear or pinion. In formulas, the number of teeth is abbreviated as "N."

Figure 19-13. The teeth of spur gears can be modified for heavier loading and higher speeds. Shown are parts made with helical gear teeth. (Boston Gear Co.)

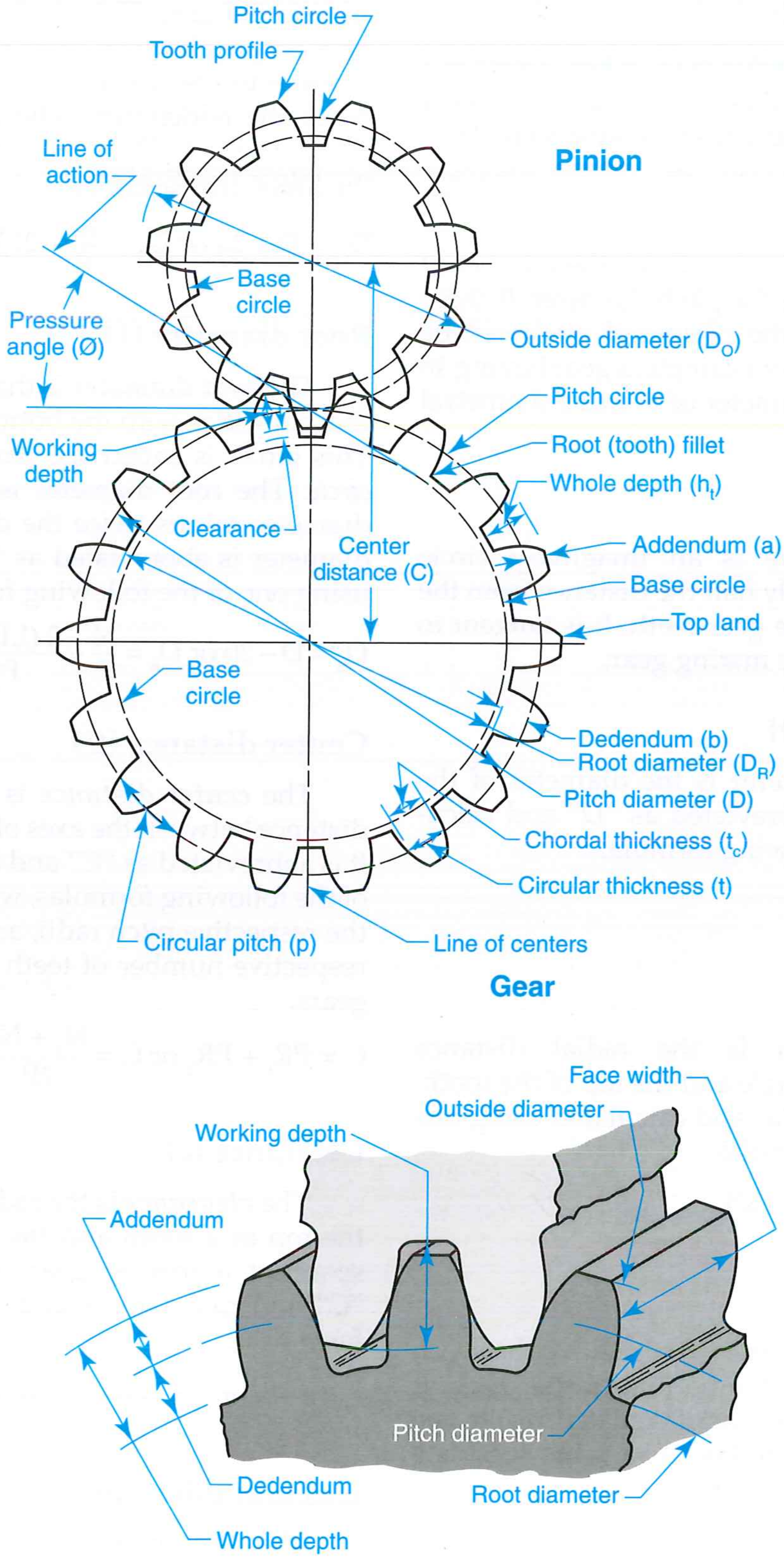

Figure 19-14. There is specific terminology that applies to spur gears. The drafter must know and understand these terms.

Diametral pitch (P)

The ***diametral pitch*** is the number of teeth in a gear per inch of pitch diameter. It is abbreviated as "P" and calculated using the following formula.

$$P = \frac{N}{D}$$

As previously discussed, N = the number of teeth. "D" represents the pitch diameter. If these values are known, the diametral pitch can be quickly calculated. For example, a gear having 48 teeth and a pitch diameter of 3" has a diametral pitch of 16.

Pitch circle

The ***pitch circle*** is an imaginary circle located approximately half the distance from the roots and tops of the gear teeth. It is tangent to the pitch circle of the mating gear.

Pitch diameter (D)

The ***pitch diameter*** is the diameter of the pitch circle. It is abbreviated as "D" and calculated using the following formula.

$$D = \frac{N}{P}$$

Addendum (a)

The ***addendum*** is the radial distance between the pitch circle and the top of the tooth. It is abbreviated as "a" and calculated using one of the following formulas.

$$a = \frac{1}{P} \text{ or } a = 0.5\,(D_O - D)$$

Dedendum (b)

The ***dedendum*** is the radial distance between the pitch circle and the bottom of the tooth. It is abbreviated as "b" and calculated using one of the following formulas. (The value 1.157 is a constant for involute gears.)

$$b = \frac{1.157}{P} \text{ or } b = 0.5\,(D - D_R)$$

Outside diameter (D_O)

The ***outside diameter*** is the diameter of a circle coinciding with the tops of the teeth of an external gear. This circle is called the *outside circle* or the *addendum circle*. The outside diameter is equal to the diameter of the pitch circle plus twice the addendum. The outside diameter is abbreviated as "D_O" and calculated using one of the following formulas.

$$D_O = D + 2a \text{ or } D_O = \frac{N}{P} + 2\left(\frac{1}{P}\right) \text{ or } D_O = \frac{N + 2}{P}$$

Root diameter (D_R)

The ***root diameter*** is the diameter of a circle that coincides with the bottom of the gear teeth. This circle is called the *root circle* or *dedendum circle*. The root diameter is equal to the pitch diameter minus twice the dedendum. The root diameter is abbreviated as "D_R" and calculated using one of the following formulas.

$$D_R = D - 2b \text{ or } D_R = \frac{N}{P} - \frac{2\,(1.157)}{P} \text{ or } D_R = \frac{N - 2.314}{P}$$

Center distance (C)

The ***center distance*** is the center-to-center distance between the axes of two meshing gears. It is abbreviated as "C" and calculated using one of the following formulas, where PR_1 and PR_2 are the respective pitch radii, and N_1 and N_2 are the respective number of teeth of the two meshing gears.

$$C = PR_1 + PR_2 \text{ or } C = \frac{N_1 + N_2}{2P}$$

Clearance (c)

The ***clearance*** is the radial distance between the top of a tooth and the bottom of the tooth space of a mating gear. It is abbreviated as "C" and calculated using one of the following formulas.

$$c = b - a \text{ or } c = \frac{1.157}{P} - \frac{1}{P} \text{ or } c = \frac{0.157}{P}$$

Circular pitch (p)

The ***circular pitch*** is the length of the arc along the pitch circle between similar points on adjacent teeth. It is abbreviated as "p" and calculated using one of the following formulas.

$$p = \frac{\pi D}{N} \text{ or } p = \frac{\pi}{P}$$

Circular thickness (t)

The ***circular thickness*** is the length of the arc along the pitch circle between the two sides of the tooth. It is abbreviated as "t" and calculated using one of the following formulas.

$$t = \frac{P}{2} \text{ or } t = \frac{\pi D}{2N}$$

Face width (F)

The ***face width*** is the width of the tooth measured parallel to the gear axis.

Chordal addendum (a_c)

The ***chordal addendum*** is the radial distance from the top of the tooth to the chord of the pitch circle. It is abbreviated as "a_c" and calculated using the following formula.

$$a_c = a + \frac{D}{2}\left[1 - \cos\left(\frac{90°}{N}\right)\right]$$

Chordal thickness (t_c)

The ***chordal thickness*** is the length of the chord along the pitch circle between the two sides of the tooth. It is abbreviated as "t_c" and calculated using the following formula.

$$t_c = D \sin\left(\frac{90°}{N}\right)$$

Whole depth (h_t)

The ***whole depth*** is the total depth of a tooth. It is equal to the addendum plus the dedendum. It is abbreviated as "h_t" and calculated using the following formula.

$$h_t = a + b \text{ or } h_t = \frac{1}{P} + \frac{1.157}{P} \text{ or } h_t = \frac{2.157}{P}$$

Working depth

The ***working depth*** is the sum of the addendums of two mating gears.

Pressure angle (ϕ)

The ***pressure angle*** is the angle of pressure between contacting teeth of meshing gears. Two involute systems, the 14 1/2° and 20° systems, are common with the standard 20° system gradually replacing the older 14 1/2° system. The pressure angle determines the diameters of the base circles of the mating gears. Referring to **Figure 19-14**, for the mating spur gears shown, the line representing the pressure angle (the ***line of action***) intersects the point of tangency between the two pitch circles (the point where the pitch circles meet). The two base circles are drawn tangent to the line of action. The term *base circle* is discussed next.

Base circle

The ***base circle*** is the circle from which the involute gear tooth profile is generated. The diameter of the base circle is determined by the pressure angle of the gear system. Referring to **Figure 19-14**, the two base circles of the mating gears are tangent to the line of action. Involute curves are developed from the surface of the base circle in order to draw the gear tooth profile. The construction of involute curves is discussed in detail in Chapter 7 of this textbook.

Spur gear representation

The normal practice in representing gears on drawings is to show the gear teeth in simplified conventional form, rather than in detailed form. See **Figure 19-15**. Drawings of gears typically show one of the views as a section view.

The circular view may be omitted unless needed for clarity. A table of gear data is included on the drawing to supply the specifications needed to manufacture the gear, **Figure 19-16**. Note that a phantom line is used to represent the outside and root diameters and a centerline is used to represent the pitch circle.

When making drawings of spur gears in this manner, manual construction procedures can be used if you are drawing manually. If you are using a CAD system, use the appropriate drawing and editing commands, object snaps, and other CAD drawing tools as needed.

Rack and Pinion Gears

A ***rack*** is a spur gear with its teeth spaced along a straight pitch line. Rack and pinion gears

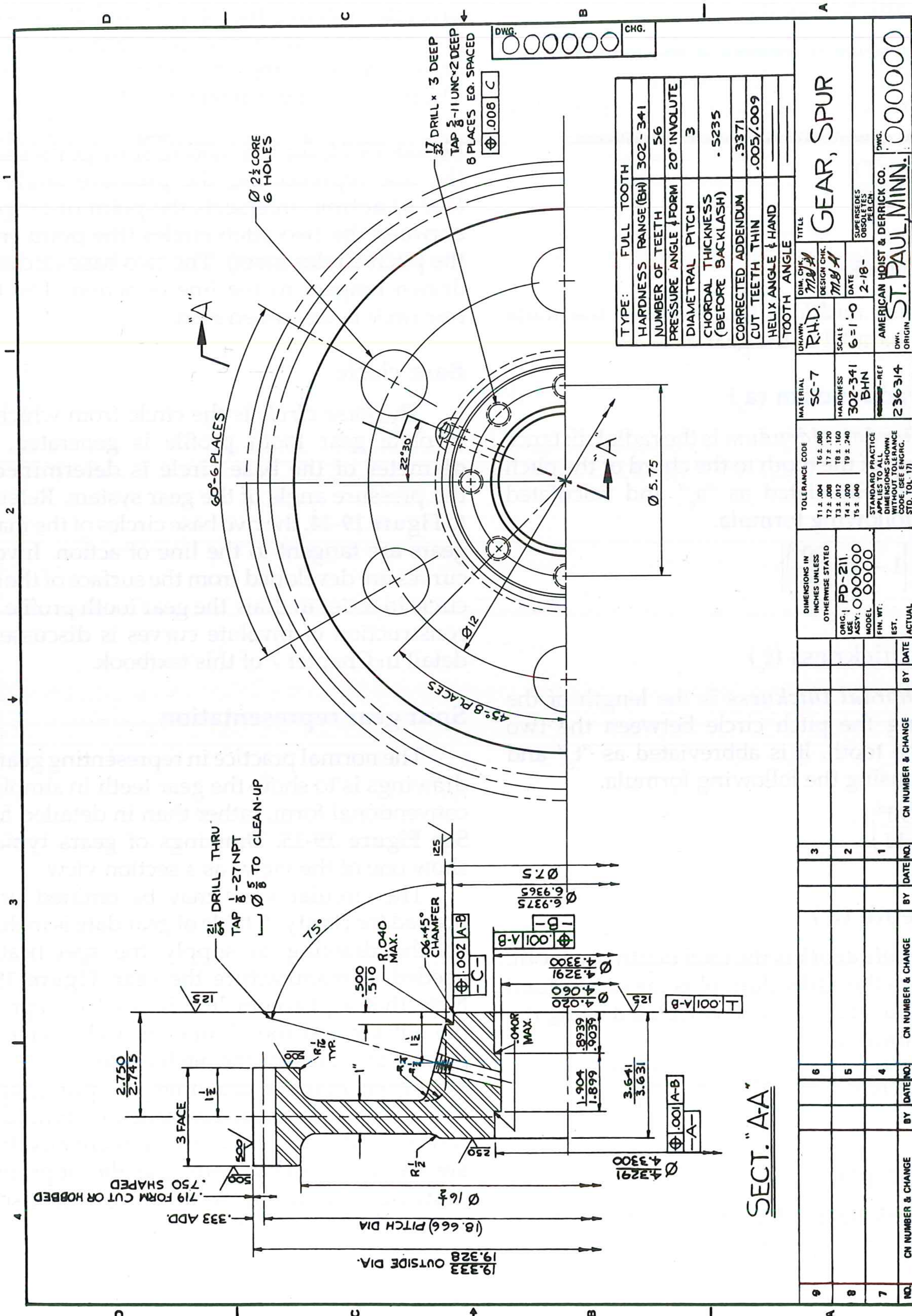

Figure 19-15. Drawings of spur gears typically show the teeth in simplified conventional form. (American Hoist & Derrick Co.)

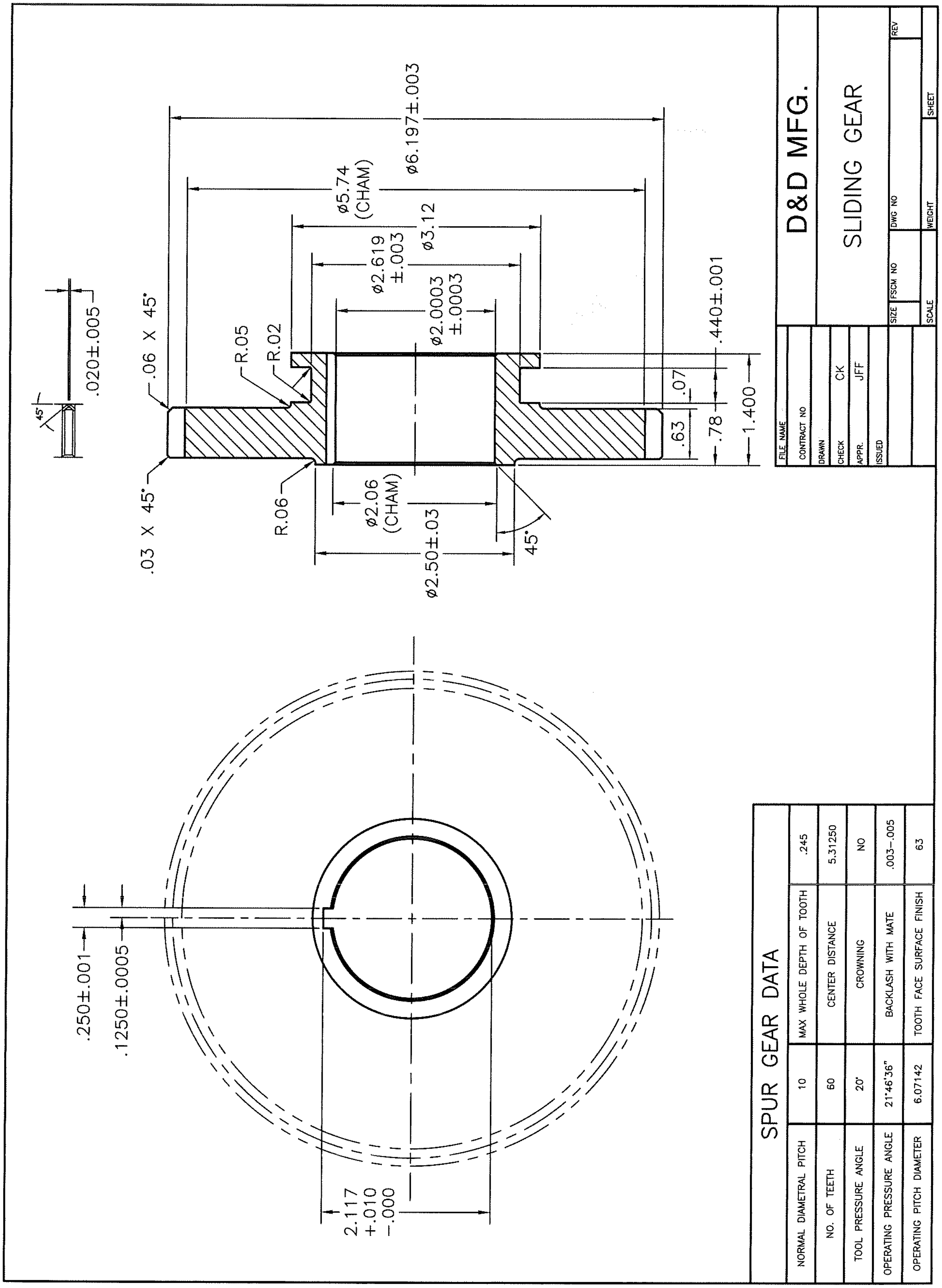

Figure 19-16. Working drawings of spur gears provide the gear data in a table on the drawing.

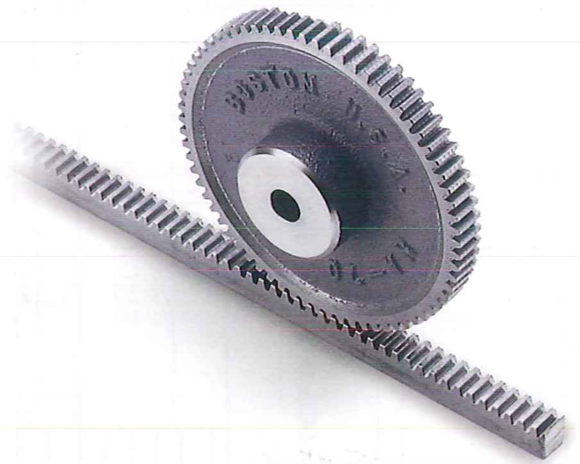

Figure 19-17. A rack is a spur gear with teeth spaced along a straight pitch line. The rack is used with a smaller gear, or pinion. (Boston Gear Co.)

are shown in **Figure 19-17**. Rack and pinion gears have a number of uses in machinery and equipment, such as lowering and raising the spindle of a drill press.

Bevel Gears

Bevel gears are used to transmit motion and power between two or more shafts whose axes are at an angle (usually 90°) and would intersect if extended, **Figure 19-18**. Bevel gears have a conical shape. Where straight bevel gears meet at 90°, two cones (each called the ***pitch cone***) meet at a common intersection point called the ***pitch apex***. Bevel gears of the same size and at 90° are called ***miter gears***. Straight bevel gears are discussed here, but helical bevel gears are often used for quieter and smoother operation.

Figure 19-18. Bevel gears are used to transmit motion between shafts that are at an angle. The smaller of the two bevel gears is called a pinion. (Boston Gear Co.)

Important terms for bevel gears are defined in the following sections and shown in **Figure 19-19**. Some of the terms used for bevel gears are the same as those used for spur gears. Formulas are given with the following terms, where appropriate, for determining straight bevel gear measurements.

Diametral pitch (P_d)

The diametral pitch for bevel gears is the same as for spur gears. It is calculated in relation to the pitch diameter.

Pitch diameter (D)

The pitch diameter is the diameter of the pitch circle at the base of the pitch cone. It is calculated using the following formula.

$$D = \frac{N}{P_d}$$

Circular pitch (p)

The circular pitch for bevel gears is the same as for spur gears.

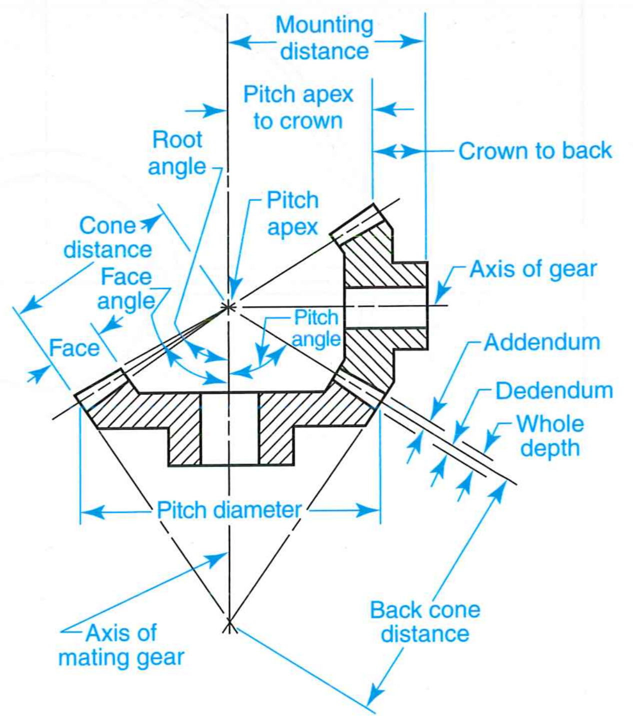

Figure 19-19. Specific terms are associated with bevel gears. The drafter must know and understand these terms.

Circular thickness (t)

The circular thickness for bevel gears is the same as for spur gears, but measured at the large end of the tooth.

Outside diameter (D_O)

The outside diameter is the diameter of the crown circle of the gear teeth. It is calculated using the following formula.

$D_O = D + 2a\cos\Gamma$

Crown height (χ)

The *crown height* is the distance from the cone apex to the crown of the gear tooth measured parallel to the gear axis.

$\chi = ½D_O/\tan\Gamma_O$

Backing (Y)

The *backing* is the distance from the back of the gear hub to the base of the pitch cone measured parallel to the gear axis.

Crown backing (Z)

The *crown backing* is the distance from the back of the gear hub to the crown of the gear, measured parallel to the gear axis.

$Z = Y + a\sin\Gamma$

Mounting distance (MD)

The *mounting distance* is the distance from a locating surface of a gear (such as the end of the hub) to the centerline of its mating gear. It is used for proper assembling of bevel gears.

$MD = Y + ½D/\tan\Gamma$

Addendum (a)

The addendum for bevel gears is the same as for spur gears, but measured at the large end of the tooth.

Addendum angle (α)

The *addendum angle* is the angle between the elements of the face cone and the pitch cone. It is the same for the gear and pinion. It is calculated using the following formula, where A_O is equal to the cone distance.

$$\alpha = \tan^{-1}\frac{a}{A_O}$$

Dedendum (b)

The dedendum for bevel gears is the same as for spur gears, but measured at the large end of the tooth.

Dedendum angle (δ)

The *dedendum angle* for bevel gears is the angle between the elements of the root cone and the pitch cone. It is the same for the gear and pinion. It is calculated using the following formula, where A_O is equal to the cone distance.

$$\delta = \tan^{-1}\frac{b}{A_O}$$

Face angle (Γ_O or γ_O)

The *face angle* is the angle between an element of the face cone and the axis of the gear or pinion. It is calculated using one of the following formulas.

Gear: $\Gamma_O = \Gamma + \delta_P$

Pinion: $\gamma_O = \gamma + \delta_G$

Pitch angle (Γ or γ)

The *pitch angle* is the angle between an element of the pitch cone and its axis. It is calculated using one of the following formulas.

Gear: $\Gamma = \tan^{-1}\dfrac{N}{n}$

Pinion: $\gamma = \tan^{-1}\dfrac{n}{N}$

Root angle (Γ_R or γ_R)

The *root angle* is the angle between an element of the root cone and the gear axis. It is calculated using one of the following formulas.

Gear: $\Gamma_R = \Gamma - \delta_G$

Pinion: $\gamma_R = \gamma - \delta_P$

Shaft angle (Σ)

The *shaft angle* is the angle between the shafts of the two gears, usually 90°.

Pressure angle (ϕ)

The pressure angle for bevel gears is the same as for spur gears.

Cone distance (A_O)

The *cone distance* is the distance along an element of the pitch cone and is the same for the gear and pinion. It is calculated using the following formula.

$$A_O = \frac{D}{2\sin\Gamma}$$

Whole depth (h_t)

The whole depth for bevel gears is the same as for spur gears, but measured at the large end of the tooth.

Chordal thickness (t_c)

The chordal thickness is the length of the chord subtending a circular thickness arc. It is calculated using one of the following formulas.

$$\text{Gear: } t_c = D\sin\left(\frac{90°\cos\Gamma}{N}\right)$$

$$\text{Pinion: } t_c = D\sin\left(\frac{90°\cos\gamma}{N}\right)$$

Chordal addendum (a_c)

The chordal addendum is the distance from the top of the tooth to the chord subtending the circular thickness arc.

$$a_c = a + \frac{D}{2\cos\Gamma}\left[1 - \cos\left(\frac{90°\cos\Gamma}{N}\right)\right]$$

Bevel gear representation

The construction of a pair of bevel gears is shown in **Figure 19-20**. As with spur gears, the gear teeth for bevel gears are normally drawn in simplified conventional form rather than in detailed form. When drawing bevel gears, manual construction procedures can be used if you are drawing manually. If you are using a CAD system, use the appropriate drawing and editing commands, object snaps, and other CAD drawing tools as needed. Begin by laying out the pitch diameters and axes of the gear and the pinion, as shown in **Figure 19-20A**. Draw light construction lines for the addendum and dedendum to show the whole tooth depth, **Figure 19-20B**. Lay off the face width and other features using the dimensions specified (or dimensions from gear data tables). Refer to **Figure 19-20C**. The completed drawing after erasing construction lines is shown in **Figure 19-20D**.

Worm Gears

Worm gears are used for transmitting motion and power between nonintersecting

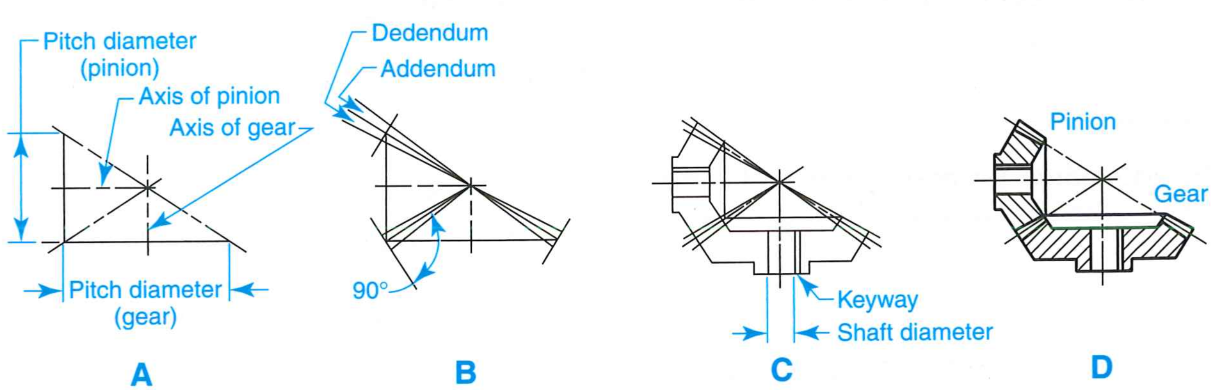

Figure 19-20. When drawing bevel gears, the teeth are usually shown in simplified conventional form.

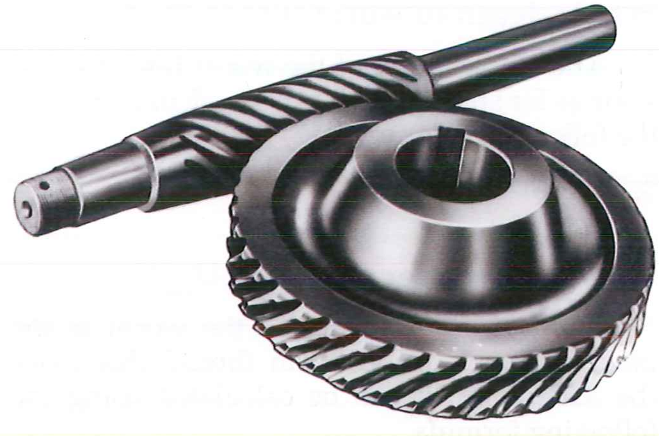

Figure 19-21. A worm gear system (worm mesh) consists of a worm and a worm gear.

shafts, usually at 90° to each other, **Figure 19-21**. A worm gear system is known as a ***worm mesh*** and consists of the *worm* and the *worm gear*. The worm is the driving member of the worm mesh. A worm mesh is characterized by a high-velocity ratio of worm to gear. Worm gears are capable of carrying greater loads than helical gears.

The worm is actually a threaded screw that appears much like a gear rack in section, **Figure 19-22**. To increase the contact of the worm mesh, the worm gear is made in a *throated* (concave) shape to wrap around the worm. One revolution of a single-threaded worm advances the worm gear one tooth space. The axial advance of the worm in one revolution is called the ***lead***. Worms can be either right-hand or left-hand thread, depending on the rotation desired.

The speed ratio of a worm mesh depends on the number of threads on the worm and the number of teeth on the gear. A worm with a single thread meshed with a gear having 48 teeth must revolve 48 times to rotate the gear one time. This is a ratio of 48:1. The same speed

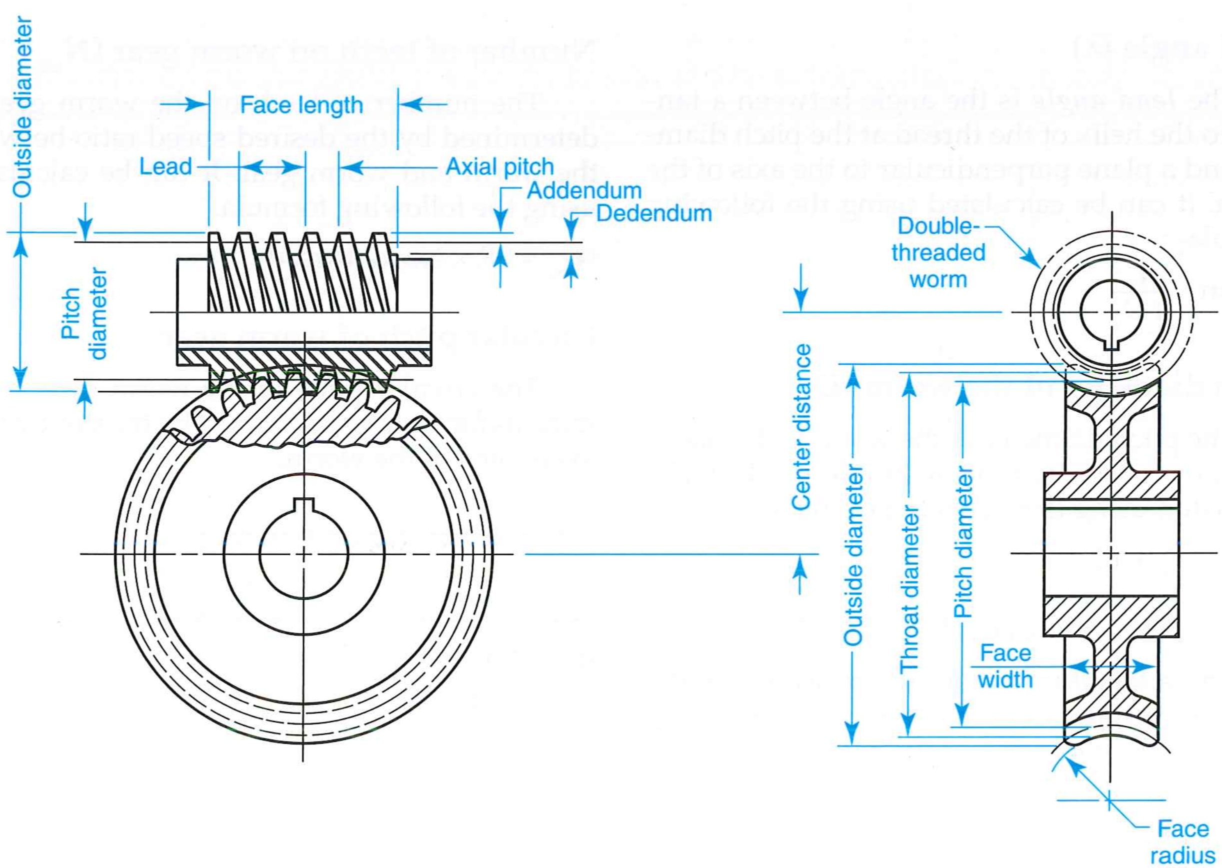

Figure 19-22. There is specific terminology that applies to worm gears. The drafter must know and understand these terms. Shown is a double-threaded worm and worm gear.

reduction with a pair of spur gears would require a gear with 480 teeth and a pinion with 10 teeth. A double-threaded worm would require 24 revolutions to rotate the 48-tooth gear once. This is a ratio of 24:1.

Important terms for worm gears are defined in the following sections. Refer to **Figure 19-22.** Formulas are given with the following terms, where appropriate, for determining worm gear measurements.

Axial pitch (P_x)

The ***axial pitch*** of the worm is the distance between corresponding sides of adjacent threads in the worm.

Lead (ι)

The ***lead*** is the axial advance of the worm in one complete revolution. The lead is equal to the pitch for single-thread worms, twice the pitch for double-thread worms, and three times the pitch for triple-thread worms.

Lead angle (λ)

The ***lead angle*** is the angle between a tangent to the helix of the thread at the pitch diameter and a plane perpendicular to the axis of the worm. It can be calculated using the following formula.

$$\lambda = \tan^{-1} \frac{\iota}{\pi D_\omega}$$

Pitch diameter of the worm (D_ω)

The pitch diameter of the worm is the diameter of the pitch circle of a worm thread. It can be calculated using the following formula.

$$D_\omega = 2.4P_x + 1.1$$

Addendum of worm thread (a_ω)

The addendum of the worm thread is the same as for spur gears. It can be calculated using the following formula.

$$a_\omega = 0.318P_x$$

Whole depth of worm thread ($h_{t\omega}$)

The whole depth of the worm thread is the same as for spur gears. It can be calculated using the following formula.

$$h_{t\omega} = 0.686P_x$$

Outside diameter of worm ($D_{O\omega}$)

The outside diameter of the worm is the pitch diameter of the worm thread plus twice the addendum. It can be calculated using the following formula.

$$D_{O\omega} = D_\omega + 0.636P_x$$

Face length of worm (F_ω)

The ***face length*** of the worm is the overall length of the worm thread section. It can be calculated using the following formula.

$$F_\omega = P_x\left(4.5 + \frac{N_{\omega G}}{50}\right)$$

Number of teeth on worm gear ($N_{\omega G}$)

The number of teeth on the worm gear is determined by the desired speed ratio between the worm and worm gear. It can be calculated using the following formula.

$$N_{\omega G} = SR \times \text{No. of threads}$$

Circular pitch of worm gear

The circular pitch of the worm gear is the same as for spur gears. It must be the same as the axial pitch of the worm.

Pitch diameter of worm gear ($D_{\omega G}$)

The pitch diameter of the worm gear is the same as for spur gears. It can be calculated using the following formula.

$$D_{\omega G} = \frac{P_x(N_{\omega G})}{\pi}$$

Addendum of worm gear ($a_{\omega G}$)

The addendum of the worm gear must equal the addendum of the worm thread. It can be calculated using the following formula.

$$a_{\omega G} = 0.318P_x$$

Whole depth of worm gear ($h_{t\omega G}$)

The whole depth of the worm gear must equal the whole depth of the worm thread. It can be calculated using the following formula.

$$h_{t\omega G} = 0.686P_x$$

Throat diameter of worm gear (D_t)

The *throat diameter* of the worm gear is the outside diameter of the worm gear measured at the bottom of the tooth arc. It is equal to the pitch diameter of the gear plus twice the addendum. It can be calculated using the following formula.

$$D_t = \frac{P_x(N_{\omega G})}{\pi} + 0.636P_x$$

Face radius of worm gear (F_r)

The *face radius* of the worm gear is the outside arc radius of the worm gear teeth that curves around the worm. It can be calculated using the following formula.

$$F_r = \frac{D_\omega}{2} - 0.318P_x$$

Outside diameter of worm gear ($D_{O\omega G}$)

The outside diameter of the worm gear is measured at the top of the tooth arc. It can be calculated using the following formula.

$$D_{O\omega G} = D_t + 0.477P_x$$

Worm gear representation

The way worm gears and worms are represented on drawings is shown in **Figure 19-23**. The gear teeth and worm thread are usually drawn in simplified, conventional form. Specifications for machining the gear and worm are given in table form on the drawing.

Splines

Splines are used to prevent rotation between a shaft and its related member, such as a coupling or a gear mounted on a shaft. A splined shaft has multiple keys similar in appearance to gear teeth around its axis. The teeth on a spline may have parallel sides or an involute profile, **Figure 19-24**.

A drawing of an external and internal spline in conventional form is shown in **Figure 19-25**. Note the specifications given for each spline on the drawing. Specific terms for involute splines are the same as those for spur gears.

Chapter Summary

Many types of machinery require mechanisms to transmit motion and power from one source to another. Cams, gears, and splines are frequently used for this purpose.

A cam is a mechanical device that changes uniform rotating motion into reciprocating motion of varying speed. Three types of cams are commonly used: the plate cam, groove cam, and cylindrical cam. A follower makes contact with the surface or groove of the cam. Uniform motion, harmonic motion, uniformly accelerated motion, and combination motion are all types of cam motion. Constructing a displacement diagram is typically the first step in designing a cam.

Gears are machine parts used to transmit motion and power by means of successively engaging teeth. Spur gears are used to transmit rotary motion between two or more parallel shafts. Bevel gears are used to transmit motion and power between two or more shafts whose axes are at an angle (usually 90°) and would intersect if extended. Worm gears are used for transmitting motion and power between nonintersecting shafts, usually at 90° to each other.

Splines are used to prevent rotation between a shaft and its related member. A splined shaft has multiple keys similar in appearance to gear teeth around its axis.

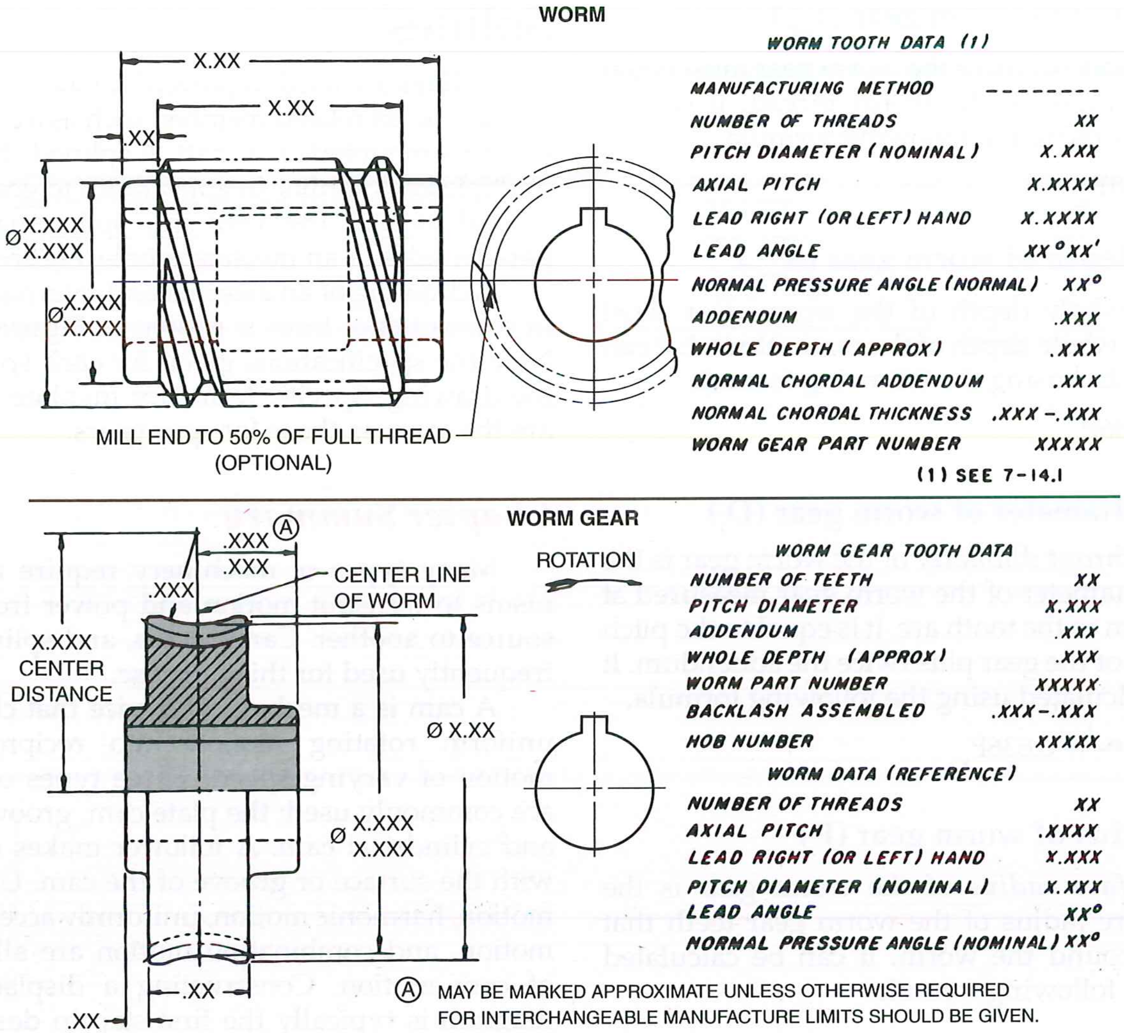

Figure 19-23. Conventions for representing and specifying worm gears and worms on drawings. (American National Standards Institute)

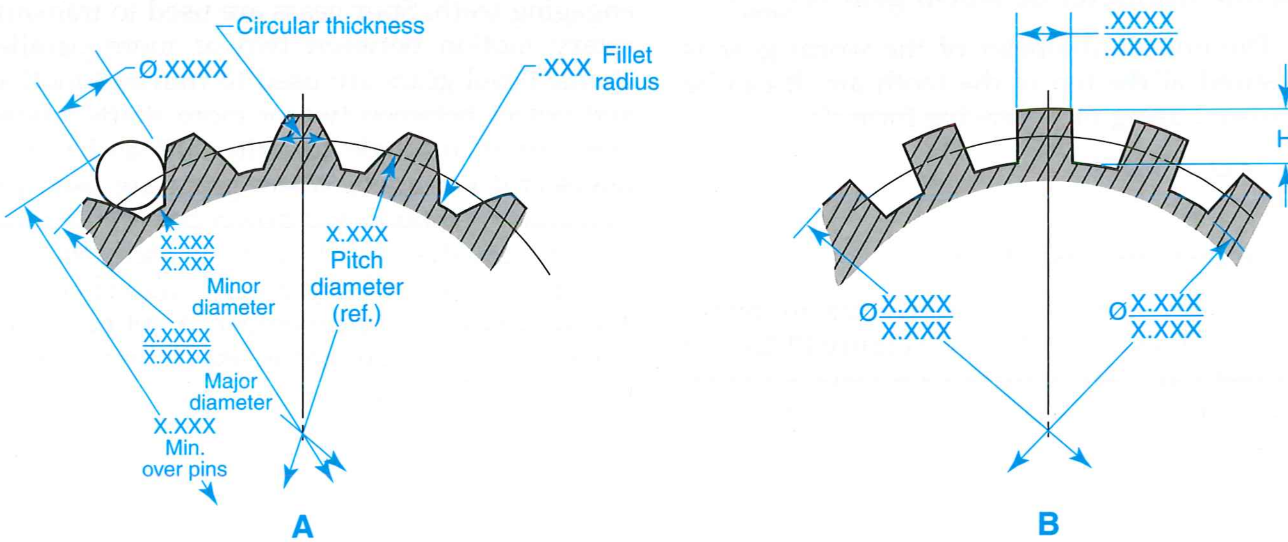

Figure 19-24. Splines are used to prevent rotary motion between a shaft and its related member. A—Involute spline. B—Parallel spline.

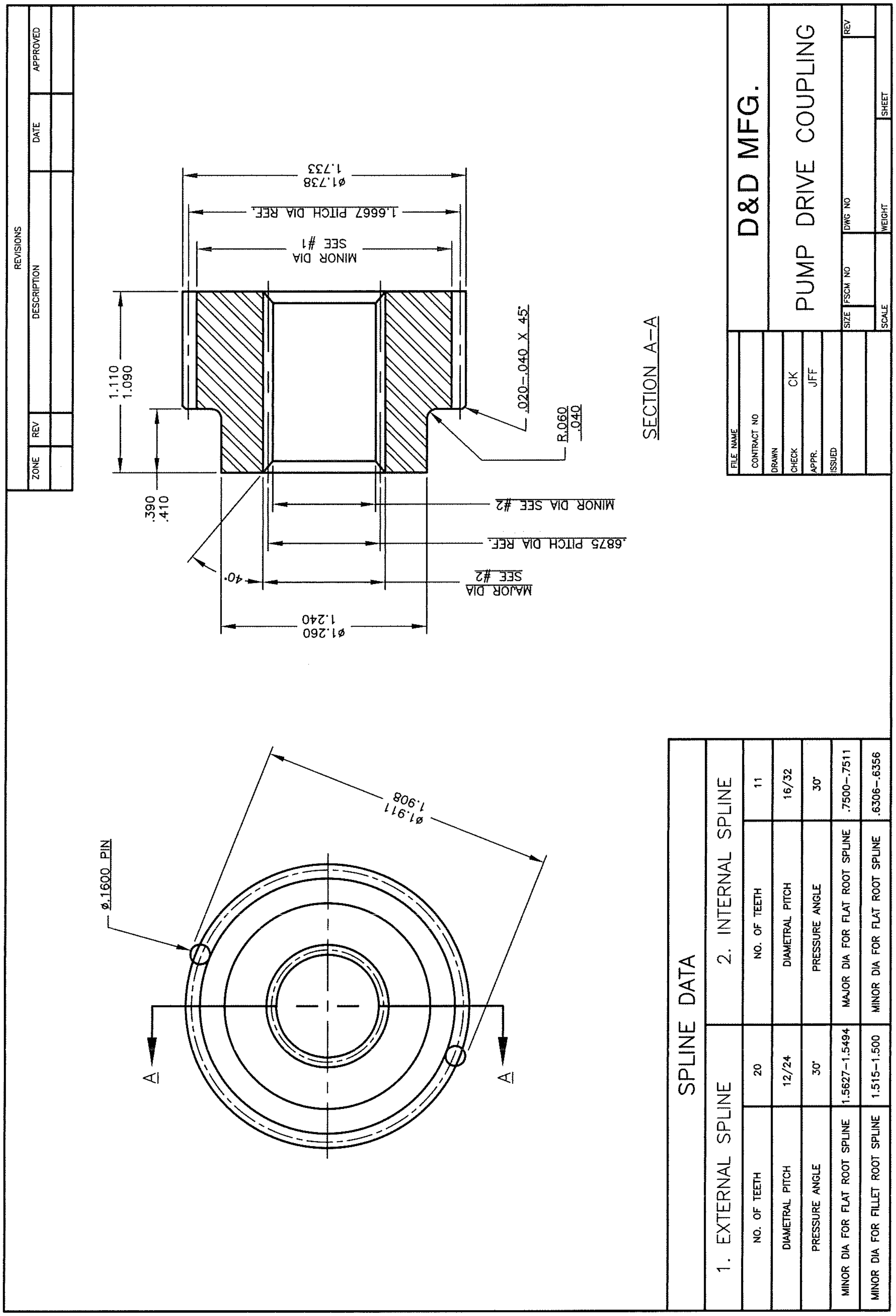

Figure 19-25. When drawing splines, the teeth are usually shown in simplified conventional form.

Review Questions

1. What is a *cam*?
2. Name the three common types of cams.
3. A _____ diagram is a graph or drawing of the travel pattern of the cam follower caused by one rotation of the cam.
4. Name the three principal types of motion for cam followers.
5. What type of cam follower motion is produced when the follower moves at the same rate of speed from the beginning to the end of the displacement cycle?
6. The length of a displacement diagram should represent _____ revolution(s) of the cam.
7. What is *dwell*?
8. What are *gears*?
9. Gear teeth designed from the _____ curve are the most commonly used teeth for gears.
10. Spur gears are used to transmit motion between two or more parallel _____.
11. When mating spur gears of different size are in mesh, the larger one is called the gear, and the smaller one is the _____.
12. The _____ is the number of teeth in a gear per inch of pitch diameter.
    A. diametral pitch
    B. outside diameter
    C. pitch diameter
    D. root diameter
13. A gear having 48 teeth and a pitch diameter of 3″ has a diametral pitch of _____.
    A. 12
    B. 14
    C. 16
    D. 18
14. The _____ is the radial distance between the pitch circle and the bottom of the tooth.
    A. addendum
    B. clearance
    C. dedendum
    D. whole depth
15. The _____ diameter is the diameter of a circle coinciding with the tops of the teeth of an external gear.
    A. minor
    B. outside
    C. pitch
    D. root
16. The _____ diameter is the diameter of a circle that coincides with the bottom of the gear teeth.
    A. minor
    B. outside
    C. pitch
    D. root
17. The _____ is the total depth of a gear tooth.
    A. circular pitch
    B. clearance
    C. whole depth
    D. working depth
18. The _____ circle is the circle from which the involute gear tooth profile is generated.
    A. addendum
    B. base
    C. outside
    D. pitch
19. The normal practice in representing gears on drawings is to show the gear teeth in _____ form, rather than in detailed form.
20. What is a *rack*?
21. _____ gears are used to transmit motion and power between two or more shafts whose axes are at an angle (usually 90°) and would intersect if extended.
22. _____ gears are used for transmitting motion and power between nonintersecting shafts, usually at 90° to each other.
23. _____ are used to prevent rotation between a shaft and its related member, such as a coupling or a gear mounted on a shaft.

Problems and Activities

The following problems provide you with an opportunity to apply the principles of designing and drawing cams and gears. These problems can be drawn manually or with a CAD system. Draw the problems as assigned by your instructor.

Use a B-size drawing sheet for each problem. Arrange the required views and other drawing elements to make good use of the space available. If you are drawing the problems manually, use one of the layout formats given in the Reference Section. If you are using a CAD system, create layers and set up drawing aids as needed. Draw a title block or use a template. Save each problem as a drawing file and save your work frequently.

Cam Layouts

The following dimensions are standard for the cam layouts in Problems 1–8. The base circle diameter is 3.50." The shaft diameter is 1", the hub diameter is 1.50", and the keyway is 1/8" × 1/16". For Problems 1–5, the knife edge follower is made from 0.625" round stock. For Problems 6–8, the roller follower diameter is 0.875". Unless otherwise noted, the follower is aligned vertically over the center of the base circle and the cam rises in 180° and falls in 180°.

1. Make a displacement diagram and cam layout for a modified uniform motion cam with a rise of 1.375". (Use an arc of one-quarter of the rise to modify the uniform motion in the displacement diagram.) The cam rotates clockwise and a knife edge follower is used.
2. Make a displacement diagram and cam layout for a modified uniform motion cam with a rise of 1.250". (Use an arc of one-third of the rise to modify the uniform motion in the displacement diagram.) The cam rotates counterclockwise and a knife edge follower is used.
3. Make a displacement diagram and cam layout for a harmonic motion cam with a rise of 1.50". The cam rotates counterclockwise and a knife edge follower is used.
4. Make a displacement diagram and cam layout for a harmonic motion cam with a rise of 1.125" in 120°, dwell for 90°, fall of 1.125" with harmonic motion in 120°, and dwell for 30°. The cam rotates clockwise and a knife edge follower is used.
5. Make a displacement diagram and cam layout for a uniformly accelerated motion cam with a rise of 1.250". The cam rotates clockwise and a knife edge follower is used.
6. Make a displacement diagram and cam layout for a uniformly accelerated motion cam with a rise of 1.375" in 90°, dwell for 90°, fall of 1.375" with uniformly decelerated motion in 90°, and dwell for 90°. The cam rotates counterclockwise and a roller follower is used.
7. Make a displacement diagram and cam layout for a uniformly accelerated motion cam with a rise of 1.125" in 120°, dwell for 60°, fall of 1.125" in 120° with uniformly decelerated motion, and dwell for 60°. The cam rotates counterclockwise and an offset roller follower is used. The roller follower is offset .50" to the left of the vertical centerline.
8. Make a displacement diagram and cam layout for a uniformly accelerated motion cam with a rise of 1.50" in 180°, dwell for 60°, and fall of 1.50" with harmonic motion in 120°. The cam rotates clockwise and has a roller follower.

Cam Design Problems

9. Design a cam that will open and close a valve on an automatic hot-wax spray at a car wash in one revolution. To open the valve, the cam follower must move 1.125". The valve is to open in 20° of cam rotation, remain open for 320°, close in 10°, and remain closed for 10°. The cam operates at moderate speed. You are to select the appropriate cam motion, base circle size, and type of cam follower. Make a full-size working drawing of the displacement diagram and the cam.
10. Design a cam that will raise a control lever, permitting a workpiece to be fed to a machine. The lever must be raised a distance of 1", remain open, and close in equal segments of cam revolution. The cam operates at a relatively high speed with

moderate pressure on the cam follower. The cam follower must be offset to the right of center .75″. Select the appropriate cam motion, base circle size, and type of cam follower. Make a full-size working drawing of the displacement diagram and the cam.

Gear Problems

11. Make a working drawing of a spur gear in simplified conventional form. Draw circular and section views. Use a shaft diameter of .75″, a hub diameter of 1.5″, a hub width of 1.00″, a face width of .50″, and a keyway with dimensions of 1/8″ × 1/16″. The gear has 40 teeth, a diametral pitch of 8, and a pressure angle of 20°. Compute values for the pitch diameter, circular thickness, and whole depth. Include the gear data in a table on the drawing.
12. Make an assembly drawing of a spur gear and pinion in simplified conventional form. Draw circular and section views. Use a shaft diameter of .625″, a face width of .75″, and a keyway with dimensions of 1/8″ × 1/16″. Where the gear and pinion meet on the drawing, draw a series of three gear teeth in detailed form. The gear has 48 teeth and a pitch diameter of 3.00″. The pinion has 24 teeth and a pitch diameter of 1.250″. The pressure angle of the system is 20°. Other dimensions of the pinion are the same as those for the spur gear. Calculate the necessary data to draw the teeth. Include the gear data in a table on the drawing.
13. Make a detail drawing of a bevel gear in simplified conventional form. Draw circular and section views. Use a shaft diameter of 1.00″, a hub diameter of 2.125″, a hub width of 1.25″, and a keyway with dimensions of 1/8″ × 1/16″. The gear has 36 teeth, a diametral pitch of 12, a pressure angle of 20°, a face width of .53″, and a mounting distance of 1.875″. Compute values for the pitch diameter, circular pitch, whole depth, addendum, and dedendum. Include the gear data in a table on the drawing.
14. Make an assembly drawing of a 64-tooth bevel gear and a 16-tooth pinion assembled at a 90° shaft angle. Draw a simplified conventional representation (section view). For the gear, the shaft diameter is .625″, the hub diameter is 2.250″, the keyway is 1/8″ × 1/16″, and the mounting distance is 1.375″. For the pinion, the shaft diameter is .375″, the hub diameter is .8125″, the keyway is 1/8″ × 3/64″, and the mounting distance is 2.50″. The diametral pitch is 16, the pressure angle is 20°, and the face width is .48″. Compute values for the pitch diameter, circular pitch, and whole depth. Include the gear data in a table on the drawing.
15. Make an assembly drawing of a worm mesh. Draw a partial conventional representation and show the gear teeth and worm thread in detailed form (refer to **Figure 19-22** in this chapter). Show the noncircular view as a section view. Use the following specifications. For the worm gear, the pitch diameter is 5.80″, the pressure angle is 20°, the number of teeth is 29, the face width is 1.375″, the shaft diameter is 1.250″, the hub diameter is 2.750″, the keyway is 1/4″ × 1/8″, and the outside diameter is 6.40″. For the worm, the pitch diameter is 2.30″, the face length is 3.0″, the shaft diameter is 1.125″, the hub diameter is 1.837″, and the keyway is 1/4″ × 1/8″.

    Note that the axial pitch of the worm can be found by computing the circular pitch of the worm gear. The other values can be found by using the formulas given in this chapter. Include the following gear data either in table form or as direct dimensions on the drawing.

    For the gear, include the number of teeth, the pressure angle, the pitch diameter, the outside diameter, and the face width. For the worm, include the lead, the pitch diameter, the outside diameter, the face length, and the whole depth of thread.
16. Design a gear assembly involving two gears, or a worm gear and a worm, to achieve a definite ratio. Obtain basic specifications for gears from a machinist's handbook or from a gear catalog. Make an assembly drawing of the gears. Add the necessary dimensions and specifications to the drawing.