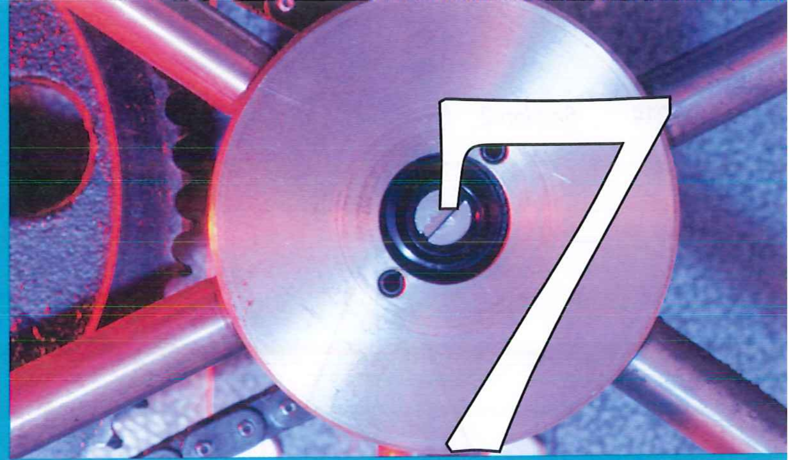
# Advanced Geometric Constructions




## Learning Objectives

After studying this chapter, you will be able to:

- Use manual drafting and CAD procedures to make geometric constructions.
- Construct ellipses.
- Construct parabolas.
- Construct hyperbolas.
- Draw special geometric curves used in drafting applications, including the spiral of Archimedes, helical curves, cycloids, and involutes.

## Technical Terms

Asymptotes
Conic sections
Construction lines
Cycloid
Directrix
Element
Ellipse
Epicycloid
Equilateral hyperbola
Foci
Focus
Helix
Hyperbola
Hypocycloid
Included angle
Involute
Lead
Major axis
Minor axis
Parabola
Pitch
Rectangular hyperbola
Spiral of Archimedes
Spline
Trammel
Transverse axis

Complex and advanced geometric constructions are difficult to solve as drafting problems. However, each problem at hand can be worked out by applying the principles of plane geometry and properly utilizing manual drafting instruments or CAD techniques.

Basic geometric problems were introduced in Chapter 6. This chapter is designed to provide specific instruction on more advanced forms of geometric construction. Some of the constructions you will study and make include conic sections, the spiral of Archimedes, the helix, cycloids, and involutes.

## Conic Sections

***Conic sections*** are curved shapes produced by passing a cutting plane through a right circular cone. A right circular cone has a circular base and an axis perpendicular to the base at its center, **Figure 7-1**. An ***element*** is a straight line drawn from any point on the circumference of the base to the peak of the cone.

Four types of curves result from orienting cutting planes at different angles. These curves are the circle, ellipse, parabola, and hyperbola, **Figure 7-2**.

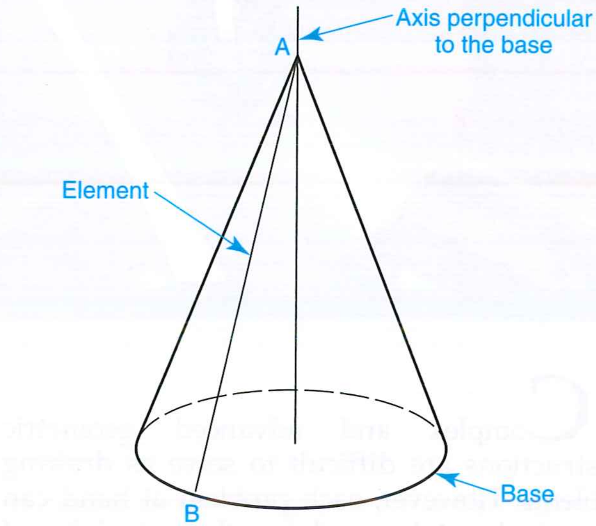



**Figure 7-1**. A right circular cone has a circular base and an axis perpendicular to the base at its center. Line AB is an element.

## *Constructing an Ellipse*

An ***ellipse*** is formed when a plane is passed through a right circular cone to make an angle with the axis greater than that of the elements. Refer to **Figure 7-2C**. An ellipse also results when a circle is viewed at an angle.

An ellipse can be defined as a curve formed by a point moving in a plane so that the sum of the distances from two fixed points is constant and equal to the major axis. The ***major axis*** is the largest diameter and the ***minor axis*** is the smallest diameter. The two fixed points are called ***foci*** (foci is the plural of *focus*).

## Construct an Ellipse Using the Foci Method

1. The major axis, Line AB, and the minor axis, Line CD, of the ellipse are given, **Figure 7-3A**.
2. Locate the foci (Points E and F) on the major axis by using a compass to strike Arcs CE and CF with radii equal to one-half the major axis.
3. On the major axis between Point E and Point O, mark points at random, **Figure 7-3B**. These points will be used to locate the ellipse. To ensure a smooth curve, space the points closely near Point E.
4. Begin construction with a point in the upper left quadrant of the ellipse. Using Points E and F as centers, and radii equal to the distance from Point A to Point Z, and from Point B to Point Z, strike intersecting arcs at Point $Z_1$.
5. Use the distance from Point A to Point Y and from Point B to Point Y as radii for intersecting arcs at Point $Y_1$. Continue in a similar manner until all points are plotted.
6. The lower left quadrant may be plotted using the same compass settings. Reverse the centers for the radii to plot the points in the two right-hand quadrants.
7. Sketch a light line through the points. Then, with the aid of an irregular curve, darken the final ellipse, **Figure 7-3C**.

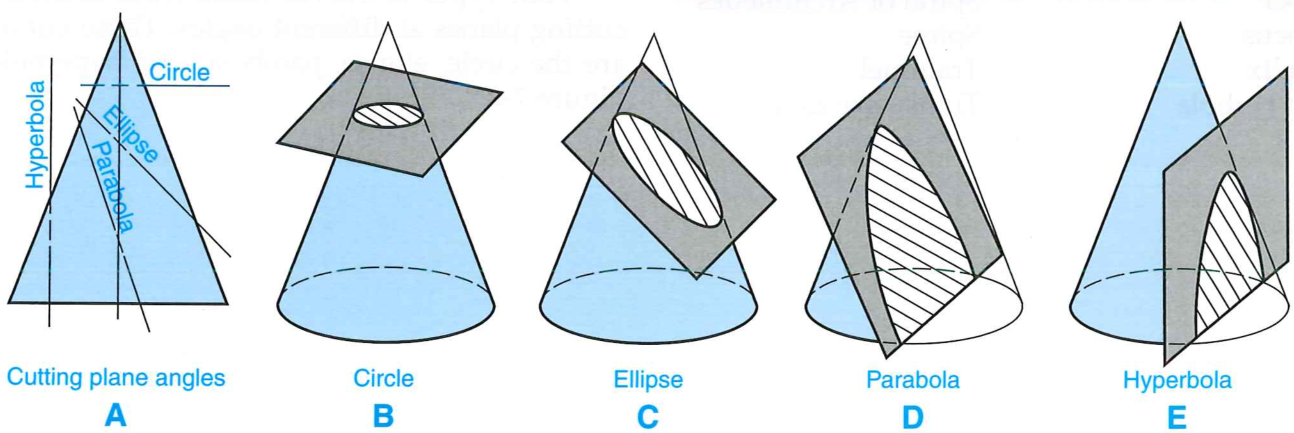



**Figure 7-2**. Conic sections are formed when a cutting plane passes through a right circular cone. A—Cutting plane angles and the resulting conic sections. B—Circle. C—Ellipse. D—Parabola. E—Hyperbola.

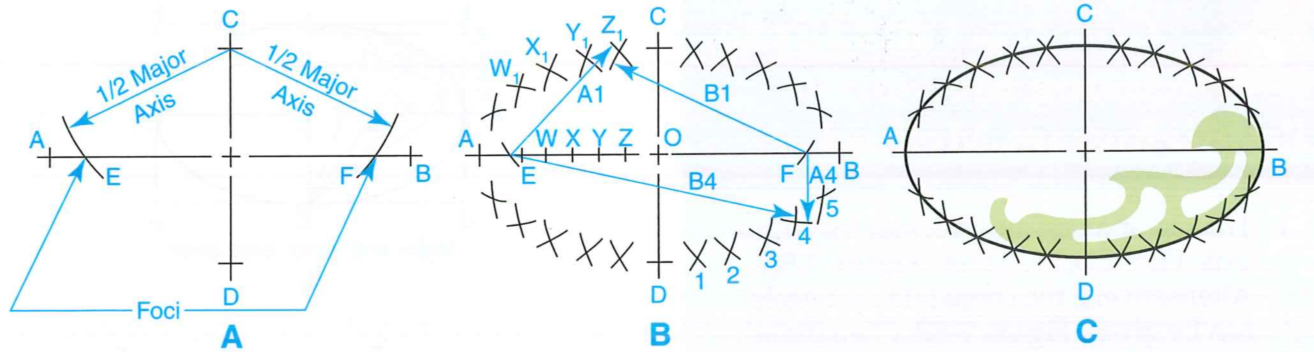



**Figure 7-3**. Constructing an ellipse using the foci method when the major and minor axes are known.

## Construct an Ellipse Using the Concentric Circle Method

1. The major axis, Line AB, and the minor axis, Line CD, are given, **Figure 7-4A**.
2. With a compass, draw circles of diameters equal to the major and minor axes.
3. Draw a diagonal, Line EE, at any point.
4. At points where the diagonal intersects with the major axis circle, draw Lines EF parallel to the minor axis. There will be two points of intersection, one in the upper left quadrant and one in the lower right quadrant.
5. At points where the diagonal intersects with the minor axis circle, draw Lines FG parallel to the major axis. The intersections of the lines at Point F are points on the ellipse curve.
6. Two additional points, Points H and J, may be located in the other two quadrants by extending Lines EF and FG.
7. Draw as many additional diagonals as needed to produce a smooth ellipse curve and project their points of intersection, **Figure 7-4B**.
8. Sketch a light line through the points. Then use an irregular curve to darken the final ellipse, **Figure 7-4C**.

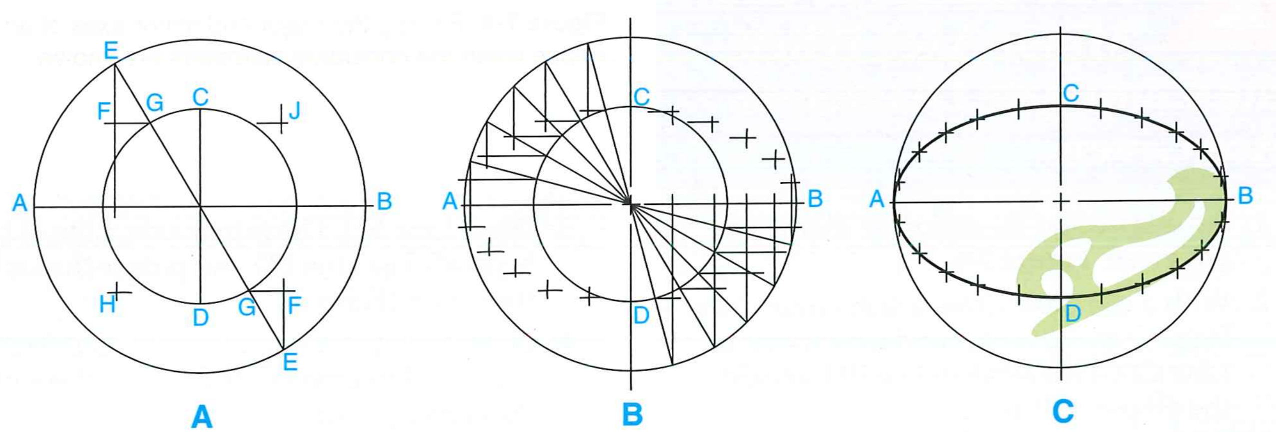



**Figure 7-4**. Constructing an ellipse using the concentric circle method when the major and minor axes are known.

## Construct an Ellipse Using the Parallelogram Method

1. The major axis, Line AB, and the minor axis, Line CD, are given, **Figure 7-5A**. Alternatively, the conjugate diameters can be given, **Figure 7-5B**. (Two diameters are conjugate when each is parallel to the tangents at the extremities of each other.) These two diameters are Lines AB and CD.
2. Construct a circumscribing parallelogram using the given axes as centerlines.
3. Divide Lines AO and AE into the same number of units. All of the units on one line should be of equal length, but the units will not necessarily be the same length as the units on the other line.
4. Draw Line DW to intersect with Line $CW_1$, Line DX to intersect with line $CX_1$, and so on. These points of intersection are plotting points for the ellipse.
5. Locate points in the remaining quadrants in a similar manner.
6. Sketch a light line through the points. Use an irregular curve to darken the final ellipse.

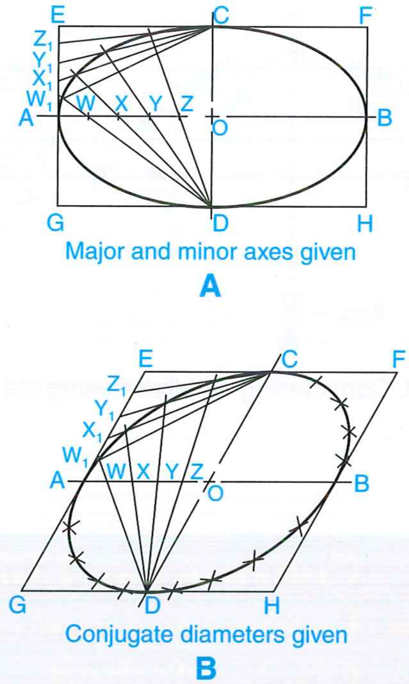



**Figure 7-5**. Constructing an ellipse using the parallelogram method. A—Drawing an ellipse when the major and minor axes are known. B—The parallelogram method can also be used when the conjugate diameters are known.

## Find the Major and Minor Axes When the Conjugate Diameters Are Given

1. An ellipse and its conjugate diameters are given, **Figure 7-6**.
2. With a compass, draw a semicircle using Point O as the center and a diameter of Line CD. This semicircle will intersect the ellipse at Point E.
3. Draw Line ED. The minor axis, Line FG, is parallel to Line ED and passes through the center (Point O).
4. Draw Line EC. The major axis, Line HJ, is parallel to Line EC and passes through the center (Point O).

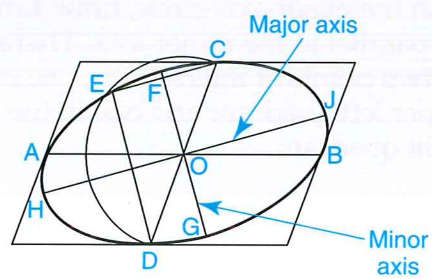



**Figure 7-6**. Finding the major and minor axes of an ellipse when the conjugate diameters are known.

## Construct an Ellipse Using the Four-Center Approximate Method

1. The major axis, Line AB, and the minor axis, Line CD, are given, **Figure 7-7A**.
2. Draw Line CB. With Point O as the center point and Radius OC, use a compass to strike an arc intersecting Line OB at Point E.
3. With Point C as the center point and Radius EB, strike an arc intersecting Line CB at Point F.
4. Construct a perpendicular bisector of Line FB. Extend this bisector to intersect with the major and minor axes at Points G and H, **Figure 7-7B**.
5. Points G and H are the centers for two of the four arcs of the ellipse. With a compass and using Point O as the center, locate Points J and K symmetrically with Points G and H.
6. Draw a line from Point H extending through Point J. Draw lines from Point K through Points J and G.
7. Using Points J and G as centers, strike Arcs JA and GB from Point $P_1$ to Point $P_2$ and from Point $P_3$ to Point $P_4$, **Figure 7-7C**.
8. With Points H and K as centers, strike Arcs HC and KD from Point $P_2$ to Point $P_3$ and from Point $P_4$ to Point $P_1$.

These four arcs will be tangent to each other, forming the four-center approximate ellipse.

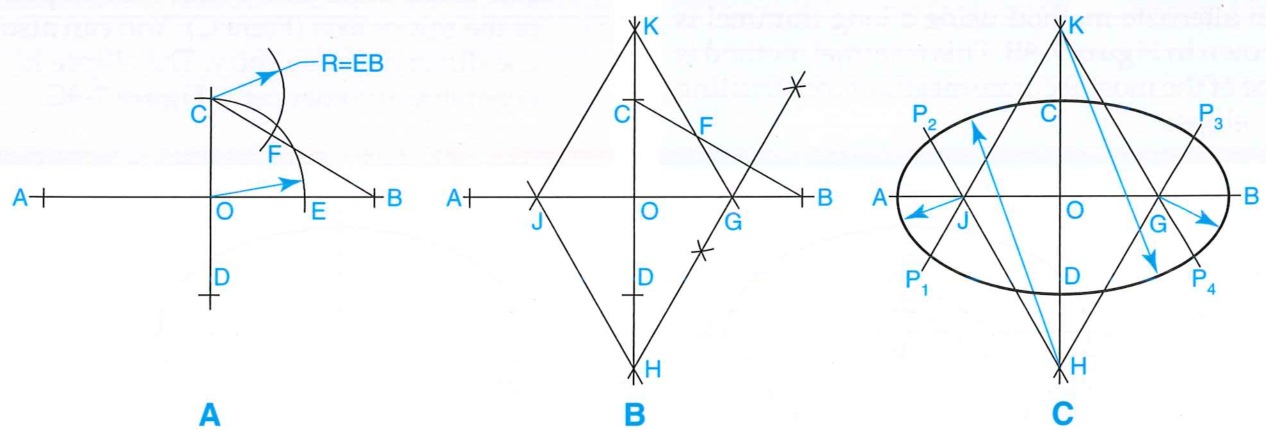



**Figure 7-7.** Constructing an ellipse using the four-center approximate method.

## Construct an Ellipse Using the Trammel Method

A ***trammel*** is an instrument used for constructing curves. Commercially produced trammels are available. You can make a trammel from anything that has a straight edge, such as a piece of paper. The following procedure uses a trammel to draw an ellipse.

1. The major axis, Line AB, and the minor axis, Line CD, are given, **Figure 7-8A**.
2. Using a piece of paper as a straightedge, lay off Points E, F, and G so that EF is equal to one-half the minor diameter (OC) and EG is equal to one-half the major diameter (OA). This marked straightedge serves as a trammel.
3. Place the trammel so that Point G is on the minor axis and Point F is on the major axis.
4. As the trammel is moved, keep Point G on the minor axis and Point F on the major axis. Point E will mark points on the ellipse. Mark enough points in each quadrant to ensure a smooth curve.
5. Sketch a light line through the points. Then, with the aid of an irregular curve, darken the final ellipse.

An alternate method using a long trammel is shown in **Figure 7-8B**. This trammel method is one of the most accurate means of constructing an ellipse.

## CAD: Draw an Ellipse Using the Ellipse Command

The **Ellipse** command greatly simplifies the task of drawing ellipses. The command provides several ways to draw an ellipse when the major and minor axes are given. The default option allows you to specify the two endpoints of one axis and an endpoint of the other axis. You can also specify the center point and two axis endpoints. The **Arc** option allows you to draw elliptical arcs by specifying start and end angles after picking the axis endpoints. To draw an ellipse with the major and minor axes given, use the following procedure.

1. The major axis, Line AB, and the minor axis, Line CD, are given. See **Figure 7-9A**.
2. Enter the **Ellipse** command. Specify the first axis endpoint by selecting the endpoint of the major axis (Point A), **Figure 7-9B**.
3. Specify the second axis endpoint by selecting the other endpoint of the major axis (Point B).
4. The next point selected specifies the distance from the midpoint of the first axis to the other axis. Select one endpoint of the minor axis (Point C). You can also use direct distance entry. The ellipse is generated automatically, **Figure 7-9C**.

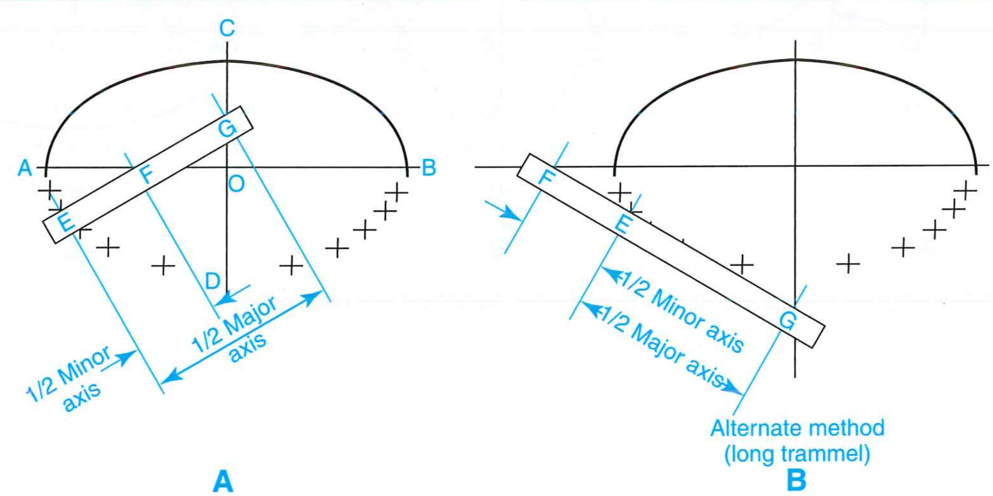



**Figure 7-8**. Constructing an ellipse using the trammel method.

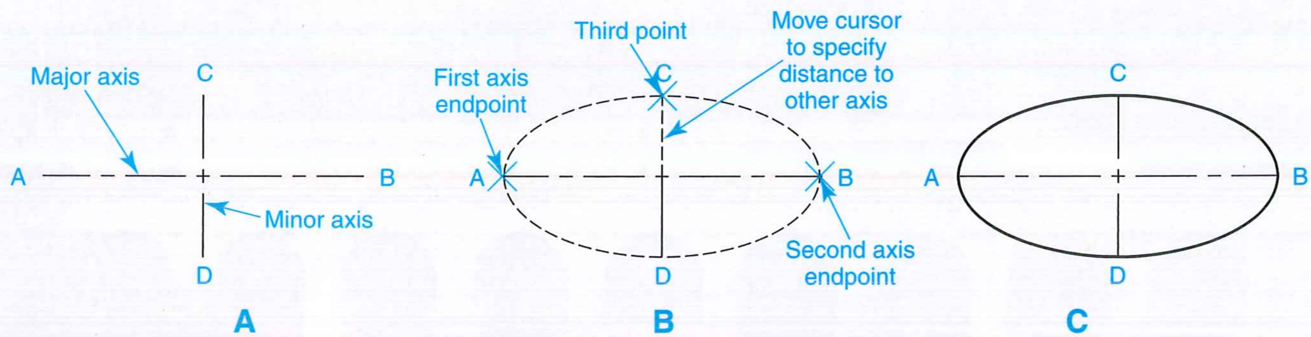



**Figure 7-9**. Using the **Ellipse** command to draw an ellipse. A—The major and minor axes are given. B—The endpoints of the major axis are selected to specify the points for the first axis. The distance to the other axis is specified by selecting an endpoint of the minor axis. C—The completed ellipse.

## Using an Ellipse Template

Considerable time can be saved in ellipse construction by using an ellipse template, **Figure 7-10**. Ellipse templates are usually designated by the ellipse angle (the angle at which a circle is viewed), **Figure 7-11**. A range of ellipse sizes is provided on each template. To use an ellipse template, line up the centerline marks on the template with the major and minor axes. This properly aligns the ellipse.

**Figure 7-10**. Templates provide a quick and easy way to construct ellipses.

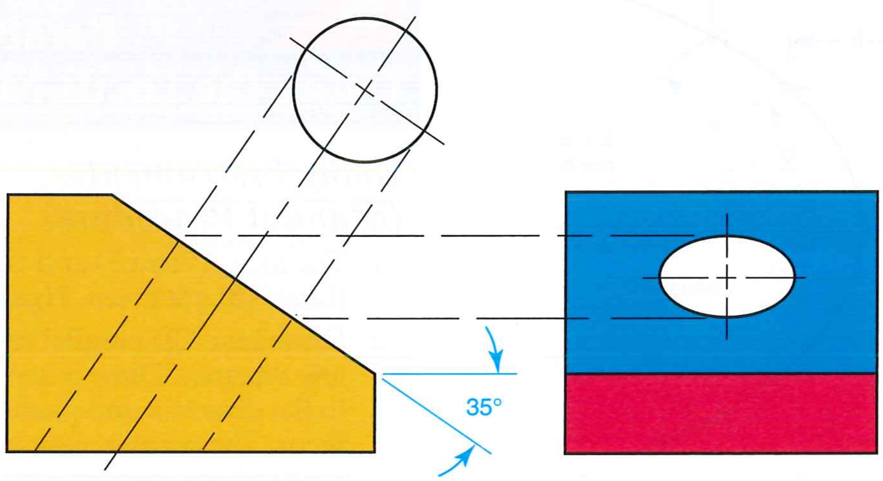



**Figure 7-11**. Ellipse templates are designated by the ellipse angle (in degrees). The ellipse angle refers to the angle at which a circle is viewed to produce an ellipse. In this example, a 35° ellipse template is required to draw the ellipse in the right-hand view.


**Figure 7-12.** Parabolic curves are commonly used in the design and construction of bridges.

## Constructing a Parabola

A ***parabola*** is formed when a plane cuts a right circular cone at the same angle as the elements. Refer to **Figure 7-2D**. The parabolic curve is used in engineering and construction for vertical curves on highway overpasses and arches on dams and bridges. See **Figure 7-12**. Parabolas are also used in forming the shape of reflectors for sound and light. Parabolas are frequently used in industrial and product design because of their pleasing appearance.

The parabola may be defined mathematically as a curve generated by moving a point so that its distance from a fixed point (the ***focus***) is always equal to its distance from a fixed line (the ***directrix***). See **Figure 7-13**.

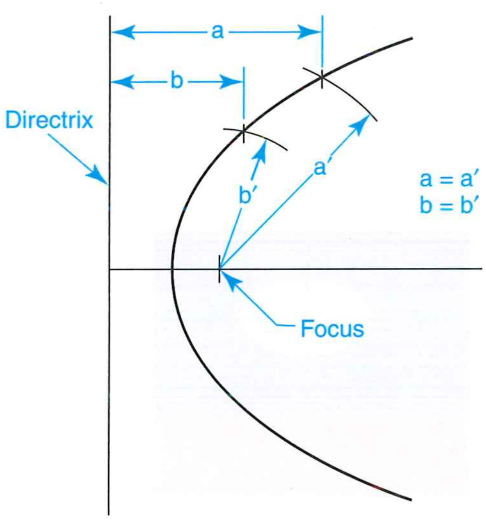



**Figure 7-13.** A parabola has a directrix and a focus. Any point on the curve is the same distance from the directrix as it is from the focus.

### CAD Construct a Parabola Using the Focus Method

#### *Using Instruments (Manual Procedure)*

1. The focus (Point F) and the directrix (Line AB) are given, **Figure 7-14**.
2. Draw Line CD parallel to the directrix at any distance. Draw a line perpendicular to the directrix and passing through the focus. The point at which this line and Line CD intersect is Point G. The point on the directrix at which the perpendicular line starts is Point E.

3. With Point F as the center and Radius EG, use a compass to strike an arc to intersect Line CD at Points H and J. These points of intersection are points on the parabola.
4. In a similar manner, locate as many points as necessary to draw the parabola.
5. Sketch a light line through the points and use the irregular curve to darken the line.
6. The vertex of the parabola (Point V) is located halfway between the origin (Point E) and the focus (Point F).

## *Using the Spline Command (CAD Procedure)*

The **Spline** command is used to draw curves called *splines.* A ***spline*** is drawn by picking points to establish the shape and direction of the curve. The resulting object is a smooth curve drawn through the points. The command works by "fitting" a curve through the points. The resulting spline can be made more accurate by editing the control points (the points controlling the shape of the curve). Existing control points can be moved, and new control points can be added. The **Spline** command is useful for drawing complex curves. To draw a parabola using the focus method, use the following procedure.

1. The focus (Point F) and the directrix (Line AB) are given. Refer to **Figure 7-14**.
2. Enter the **Offset** command and offset Line AB at any distance to draw Line CD.
3. Enter the **Line** command. Draw a line from the focus (Point F) perpendicular to Line AB. Use object snaps as needed. The point at which this line and Line AB intersect is Point E. Then, enter the **Line** command and draw a line from Point F past Line CD. Use Ortho mode. The point at which this line and Line CD intersect is Point G.
4. Enter the **Circle** command. With Point F as the center and Radius EG (the offset distance between Lines AB and CD), draw a circle to intersect Line CD at Points H and J. These points of intersection are points on the parabola.
5. Enter the **Circle** command and draw circles to locate as many points as necessary to draw the parabola.
6. Enter the **Spline** command. Draw the curve by picking the intersection points. Begin with the lower half of the curve and pick points in a counterclockwise manner starting with the point furthest away from Point V (the vertex). Work toward the vertex and complete the curve by picking the vertex and the points defining the upper half of the curve. After picking the final point along the upper half of the curve, press [Enter] twice to accept the default start and end tangents. The start and end tangents establish the tangent directions of the curve at the start and end points.
7. If necessary, enter the **Splinedit** command to modify the shape of the curve to pass through the intersection points. When you enter this command and pick the spline, the control points are highlighted. Use the **Fit data** option to move or add control points if needed.

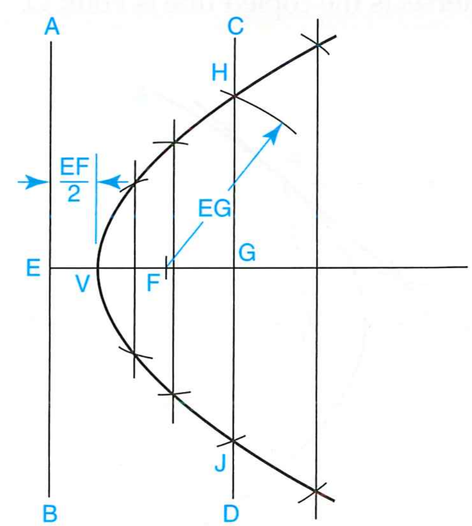



**Figure 7-14.** Using the focus method to construct a parabola when the directrix and the focus are known.

## Construct a Tangent to a Parabola

### *Using Instruments (Manual Procedure)*

1. Parabola AB, its axis (Line CD), the focus (Point F), and the point of tangency (Point P) are all given, **Figure 7-15**.
2. Draw Line PO parallel to the axis and Line PF through the focus. Bisect Angle OPF. The bisector (Line PQ) is tangent to the parabola at Point P.

### *Using the Line and Circle Commands (CAD Procedure)*

1. Parabola AB, its axis (Line CD), the focus (Point F), and the point of tangency (Point P) are all given. Refer to **Figure 7-15**.
2. Enter the **Copy** command and copy Line CD. Select Point D as the base point and Point P as the second point. Enter the **Line** command and draw Line PF using object snaps.
3. Enter the **Circle** command. Select Point P as the center and select Point F to specify the radius. The point where the circle intersects the copied line is Point O.
4. Enter the **Line** command and draw a line to bisect Angle OPF. Specify Point P as the first point. To specify the second point, enter the Mid Between Points object snap and select Points O and F. The bisector (Line PQ) is tangent to the parabola at Point P.

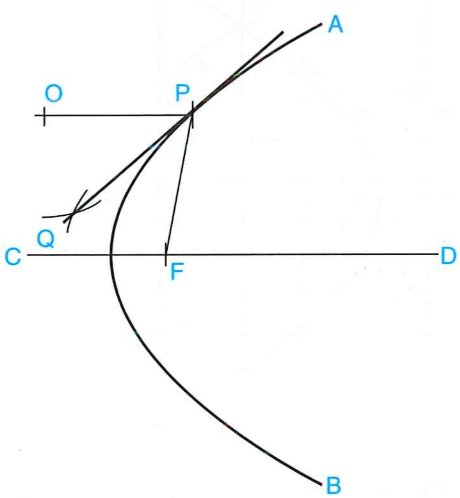



**Figure 7-15**. A tangent to a given parabola can easily be constructed at any given point.

## Construct a Parabola Using the Tangent Method

1. Points A and B, and the distance from Line AB to the vertex (Point D) are given, **Figure 7-16**.
2. Extend Line CD to Point E so that Line DE is equal to Line CD.
3. Draw Lines AE and BE. These lines are tangent to the parabola at Points A and B.
4. Divide Lines AE and BE into the same number of equal parts. The accuracy of this construction increases with the number of divisions. Number the points from opposite ends.
5. Draw lines between the corresponding numbered points.
6. These lines are tangent to the required parabola.
7. Sketch a light line tangent to these lines. Use an irregular curve to darken the lines.

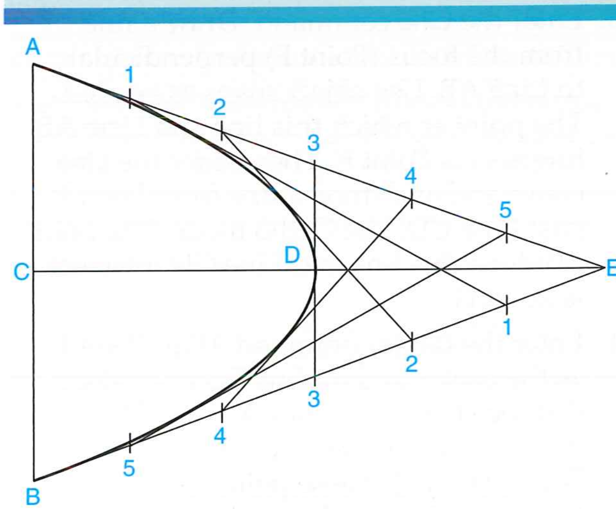



**Figure 7-16**. Using the tangent method to construct a parabola when the endpoints and the distance from a line drawn between them and the vertex are known.

## CAD Construct a Parabola Through Three Given Points Using the Spline Command

1. Points A and B and the vertex, Point D, are given. Refer to **Figure 7-16**.
2. Enter the **Spline** command. Draw a spline through Points B, D, and A. Use the Endpoint object snap. Specify the default start and end tangents. The resulting spline is a smooth curve passing through the three points.

## CAD Construct a Parabolic Curve Through Two Given Points

### *Using Instruments (Manual Procedure)*

1. Points A and B are given, **Figure 7-17**.
2. Choose any point as Point C and draw two tangent lines, Lines CA and CB.
3. Construct the parabolic curve using the tangent method. Refer to **Figure 7-16** for an example of this construction.

Note that the distances of Lines CA and CB are not necessarily equal. Refer to **Figure 7-17B** and **Figure 7-17C**. When these two distances are equal, the bisector of Angle ACB is also the axis of the parabola. Refer to **Figure 7-17A**.

### *Using the Spline Command (CAD Procedure)*

1. Points A and B are given. Refer to **Figure 7-17A**.
2. Enter the **Line** command and draw Lines CA and CB. Choose any point as Point C.
3. Enter the **Line** command and draw a line from the midpoint of Line CA to the midpoint of Line CB. Use the Midpoint object snap. Enter the **Line** command again and draw a line from Point C to bisect this line. Use object snaps. The intersection of the two lines is the vertex of the parabola.
4. Enter the **Spline** command and draw a three-point spline. Specify Point B as the first point, the vertex as the second point, and Point A as the third point. The resulting spline is a smooth curve passing through the three points.

If the distances of Lines CA and CB are not equal, use the **Divide** command to divide the two lines into the same number of equal parts and use the **Line** command to draw lines connecting the division points. When drawing the lines, use the Node object snap to snap to the division points. Refer to **Figure 7-17B** and **Figure 7-17C**. Use the **Spline** command to draw the curve by picking points at locations where tangent points occur. Use object snaps as needed. If it is necessary to increase the accuracy of the spline, use the **Splinedit** command.

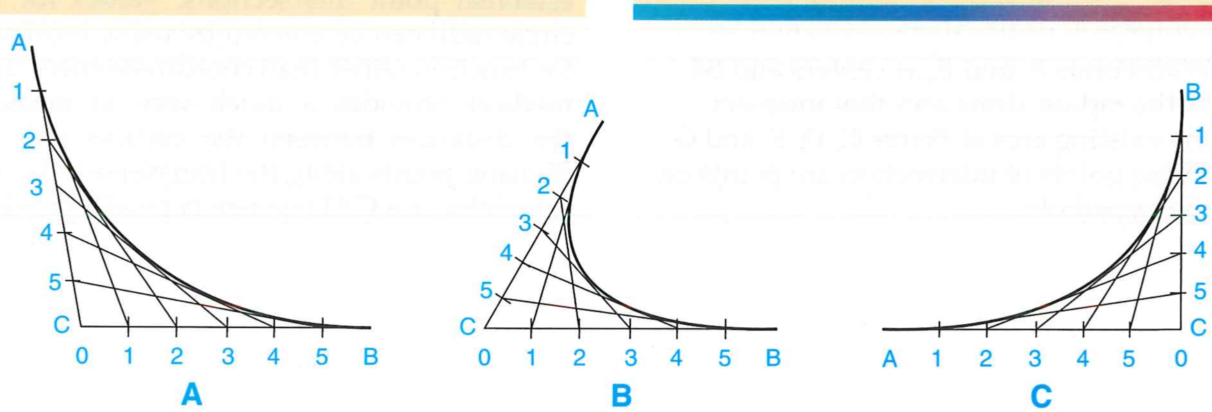



**Figure 7-17.** Using the tangent method to construct a parabola when two points are given (using an assumed third point).

## Constructing a Hyperbola

A ***hyperbola*** is formed when a plane cuts two right circular cones that are joined at their vertices, **Figure 7-18**. Mathematically, a hyperbola is defined as a plane curve traced by a point moving so that the difference of its distance from two fixed points (the foci) is a constant equal to the transverse axis. The ***transverse axis*** is the distance between the vertices of the two curves. The ***asymptotes*** of the hyperbola are lines that intersect at the midpoint of the transverse axis. The lines of the hyperbola will approach, but not intersect, the asymptotes if extended to infinity.

Hyperbolic curves are used in space probes. The equilateral hyperbola can be used to indicate varying pressure of gas as the volume varies. Gas pressure varies inversely as the volume changes.

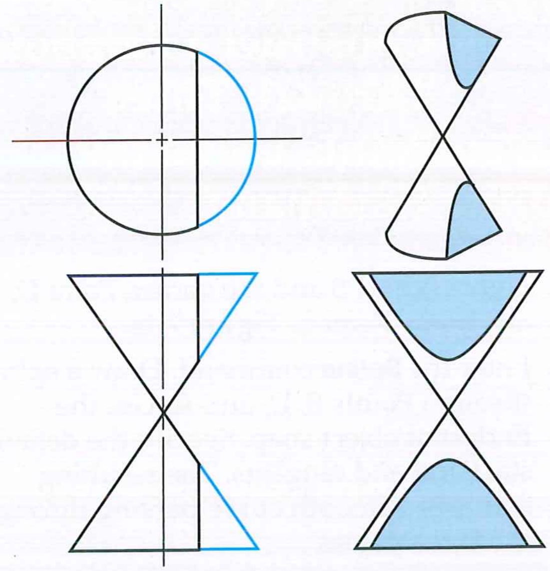


**Figure 7-18.** A hyperbola is formed when a plane passes through two right circular cones that are joined at their vertices.

## Construct a Hyperbola Using the Foci Method

### *Using Instruments (Manual Procedure)*

1. The foci, Points $F_1$ and $F_2$, and the transverse axis, Line AB, are given, **Figure 7-19**.
2. Lay off a convenient number of points to the right of Point $F_2$.
3. With Points $F_1$ and $F_2$ as centers and A4 (in the example) as the radius, use a compass to draw Arcs C, D, E, and G.
4. With Points $F_1$ and $F_2$ as centers and B4 as the radius, draw arcs that intersect the existing arcs at Points C, D, E, and G. These points of intersection are points on the hyperbola.
5. Continue to lay off intersecting arcs, using radii of A1 and B1, A2 and B2, and so on.
6. Sketch a light line through the points. Use an irregular curve to darken the final curve.

### *Using the Spline Command (CAD Procedure)*

To draw a hyperbola, you can use the **Spline** command to draw a curve through a series of plotted points. To locate the points, use the **Circle** command and draw circles to establish point intersections. Values for the circle radii can be entered by using a calculator function rather than coordinate entry. This method provides a quick way to measure the distances between the vertices and the division points along the transverse axis. The calculator of a CAD system typically provides a number of functions to determine linear distances. Normally, you can access the system calculator during a command sequence. The following procedure uses the **Distance Between Two Points** function to calculate distances for user input.

1. The foci, Points $F_1$ and $F_2$, and the transverse axis, Line AB, are given. Refer to **Figure 7-19**.
2. Enter the **Line** command. Draw a line from Point $F_2$ to a point you will use as the first division point on the transverse axis. Use Ortho mode and direct distance entry. Enter the **Line** command again and draw a horizontal line to the right of this line at a convenient length. Enter the **Divide** command and divide this line into a convenient number of points (such as 4). Refer to **Figure 7-19**.
3. Enter the **Circle** command. Specify Point $F_1$ as the center. To specify the radius, access the system calculator and enter the **Distance Between Two Points** function. Using object snaps, pick Point A and the fourth division point to calculate Radius A4. Specify this value as the radius of the circle.
4. Enter the **Copy** command. Copy the circle by specifying Point $F_1$ as the base point and Point $F_2$ as the second point.
5. Enter the **Circle** command. Specify Point $F_2$ as the center. Specify Radius B4 by using the **Distance Between Two Points** function.
6. Enter the **Copy** command. Copy the circle drawn in Step 5 by specifying Point $F_2$ as the base point and Point $F_1$ as the second point.
7. The points of intersection established by the circle intersections are points on the hyperbola. Continue drawing intersecting circles using radii of A1 and B1, A2 and B2, and so on.
8. Enter the **Spline** command. Draw Hyperbola CBD by picking the intersection points beginning with Point C and working downward. Specify the default start and end tangents. Enter the **Spline** command again and draw Hyperbola GAE by picking the intersection points beginning with Point E and working upward. Specify the default start and end tangents.

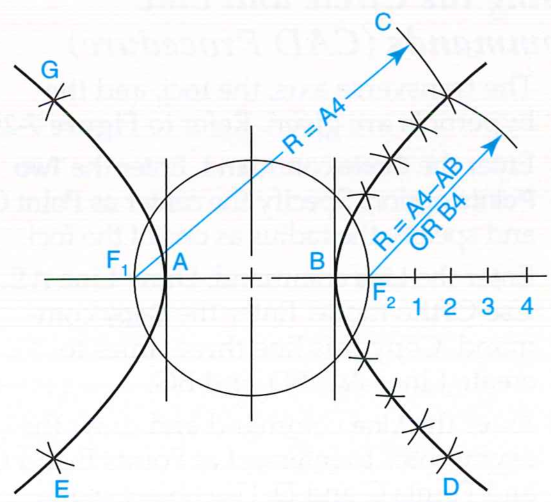



**Figure 7-19.** Using the foci method to construct a hyperbola when the foci and the transverse axis are given.

## Locate the Asymptotes of a Hyperbola

### *Using Instruments (Manual Procedure)*

1. The transverse axis, the foci, and the hyperbola are given, **Figure 7-20**.
2. With a compass, draw a circle with the center at the midpoint of the transverse axis (Point O) and passing through the foci.
3. Construct perpendiculars to the transverse axis at Points A and B. These points are the vertices of the hyperbola.
4. The asymptotes extend through the points where the perpendiculars intersect the circle.

### *Using the Circle and Line Commands (CAD Procedure)*

1. The transverse axis, the foci, and the hyperbola are given. Refer to **Figure 7-20**.
2. Enter the **Circle** command. Enter the **Two Points** option. Specify the center as Point O and specify the radius as one of the foci.
3. Enter the **Line** command. Draw Line AE. Use Ortho mode. Enter the **Copy** command. Copy this line three times to create Lines AC, BD, and BG.
4. Enter the **Line** command and draw the asymptotes to intersect at Points E and G and Points C and D. Use object snaps.
5. To extend the asymptotes past the intersection points on the circle, enter the **Circle** command and draw a larger circle with the center at Point O. Drag the cursor to specify a radius large enough to provide a boundary edge for extending the lines to the desired length.
6. Enter the **Extend** command. Select the larger circle as the boundary edge. Extend Lines CD and EG to the circumference of the circle by selecting each line twice (select points near the two endpoints of each line).

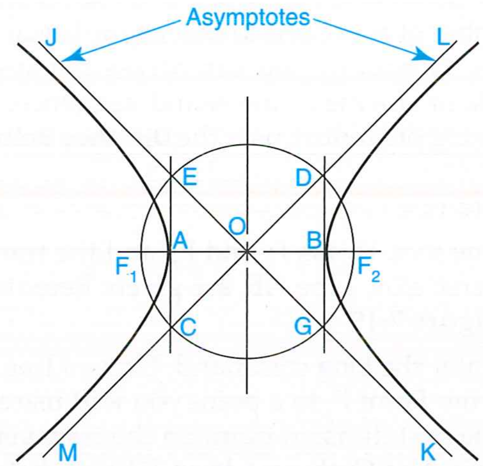



**Figure 7-20.** Locating the asymptotes of a hyperbola.

## Construct a Tangent to a Hyperbola

### *Using Instruments (Manual Procedure)*

1. Hyperbola LBK, the point of tangency (Point P), and the foci are given, **Figure 7-21**.
2. Draw lines from Point P to the foci.
3. Bisect Angle $F_1PF_2$. The bisector (Line HP) is the required tangent.

### *Using the Line and Circle Commands (CAD Procedure)*

1. Hyperbola LBK, the point of tangency (Point P), and the foci are given. Refer to **Figure 7-21**.
2. Enter the **Line** command. Draw lines from Point P to the foci. Use object snaps as needed.
3. Bisect Angle $F_1PF_2$. Enter the **Circle** command and specify the center as Point P. Drag the cursor to specify the radius at a convenient distance. Refer to the arc segments intersecting Lines $PF_1$ and $PF_2$ in **Figure 7-21**. Enter the **Circle** command again and draw another circle. Locate the center where the previously drawn

circle intersects Line $PF_1$. Specify the radius as slightly greater than one-half the radius of the previously drawn circle. Enter the **Copy** command and copy this circle. Locate the center where the first circle drawn intersects Line $PF_2$. The intersection of the two circles is the point of intersection for the tangent line. Refer to the intersecting arcs in **Figure 7-21**.

4. Enter the **Line** command and draw the bisector (Line HP). This line is the required tangent.

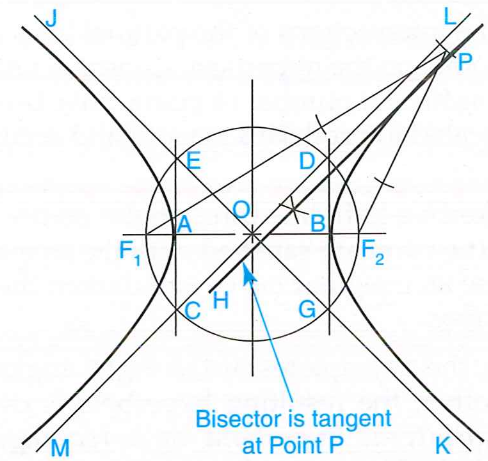



**Figure 7-21**. Drawing a tangent to a point on a hyperbola when the foci are given.

## CAD Construct a Hyperbola with the Asymptotes and One Point on the Curve Given

### *Using Instruments (Manual Procedure)*

1. The asymptotes (Lines OA and OB) and Point P on the curve are given, **Figure 7-22A**.
2. Through Point P, draw Lines CD and EG parallel to the asymptotes.
3. From the origin (Point O), draw radial lines intersecting Line CD at Points 1, 2, 3, 4, and 5, and Line EG at Points 1′, 2′, 3′, 4′, and 5′.
4. Draw lines parallel to the asymptotes at Points 1 and 1′, 2 and 2′, and so on. Draw lines parallel to the asymptotes where the radial lines intersect Lines EG and CD.

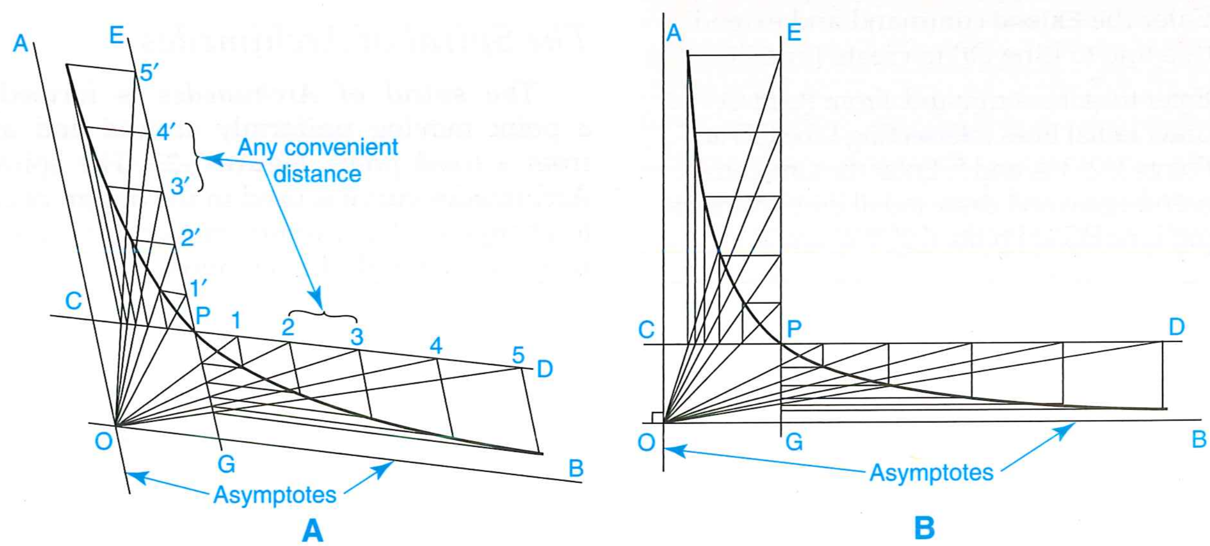



**Figure 7-22**. Constructing a hyperbola when the asymptotes and a point on the curve are given. A—Radial lines from the origin and lines parallel to the asymptotes are drawn to establish points on the curve. B—If the asymptotes form a right angle, the hyperbola is an equilateral hyperbola (also called a rectangular hyperbola).

5. The intersections of the parallel lines are points on the hyperbola. Continue until a sufficient number of points have been located to produce a smooth and accurate curve.
6. Sketch a light line through the points. When you are satisfied with the shape, use an irregular curve and darken the curve.

When the asymptotes are at right angles to each other, the resulting hyperbola is called an ***equilateral hyperbola*** or a ***rectangular hyperbola,*** **Figure 7-22B**.

### Using the Line and Spline Commands (CAD Procedure)

1. The asymptotes (Lines OA and OB) and Point P on the curve are given. Refer to **Figure 7-22A**.
2. Enter the **Line** command. From Point P, draw a line parallel to Line OB. Use the Parallel object snap and draw the line at a distance approximately equal to PD. Enter the **Extend** command and extend this line to Line AO to create Line CD.
3. Enter the **Line** command. From Point P, draw a line parallel to Line AO. Use the Parallel object snap and draw the line at a distance approximately equal to PE. Enter the **Extend** command and extend this line to Line OB to create Line EG.
4. Enter the **Line** command. From Point O, draw radial lines intersecting Line CD at Points 1, 2, 3, 4, and 5. Enter the **Line** command again and draw radial lines intersecting Line EG at Points 1′, 2′, 3′, 4′, and 5′.
5. Enter the **Line** command and draw lines parallel to the asymptotes at Points 1 and 1′, 2 and 2′, and so on. Use the Parallel object snap. Enter the **Line** command again and draw lines parallel to the asymptotes where the radial lines intersect Lines EG and CD. Use the Parallel object snap.
6. The intersections of the parallel lines are points on the hyperbola. Additional radial and parallel lines can be drawn to create intersection points and make the curve longer.
7. Enter the **Spline** command. Starting at the uppermost intersection point, pick the points to define the curve. Specify the default start and end tangents to complete the curve.

## Constructing Other Curves

Other curves commonly used in engineering, design, and drafting are the spiral of Archimedes, the helix, the cycloid, and the involute. Construction procedures and applications for these curves are discussed in the following sections.

### The Spiral of Archimedes

The ***spiral of Archimedes*** is formed by a point moving uniformly around and away from a fixed point, **Figure 7-23**. The spiral of Archimedes curve is used in the design of cams to change uniform rotary motion into uniform reciprocal (straight-line) motion.

# Construct a Spiral of Archimedes

## *Using Instruments (Manual Procedure)*

1. The rise of one revolution, OB, is given. Refer to **Figure 7-23**.
2. Draw a horizontal line (Line AB) through Point O. Lay off a convenient number of equal parts totaling 1 1/2″ (for example, 12 parts of 1/8″ each) on Line OB (Line OB is 1 1/2″ long).
3. With Point O as the center and Radius OB, use a compass to draw a circle.
4. Divide the circle into the same number of equal parts as Line OB (in this example, 12 equal parts of 30° each). Number each line, starting with the first line after Line OB.
5. With Point O as the center, draw an arc with a radius equal to the distance from Point O to equal part 1. The arc should start on Line OB and end on Line 1.
6. Continue with concentric arcs for each of the equal parts. Start the second arc on Line OB and end it on the corresponding numbered line (the arc starting on the second equal part will end on Line 2, the arc starting on the seventh equal part will end on Line 7, and so on).
7. The points of intersection of the concentric arcs and radial lines are points on the spiral curve.
8. Sketch a light line through these points. Finish with an irregular curve.

## *Using the Circle and Spline Commands (CAD Procedure)*

1. The rise of one revolution, OB, is given. Refer to **Figure 7-23**.
2. Enter the **Divide** command. Divide Line OB into 12 equal parts.
3. Enter the **Array** command. Select Line OB and create a polar array. Specify the number of objects as 12 and specify the center point as Point O. Specify an angular rotation of 360° and rotate the objects as they are copied. This creates a pattern of lines offset at 30° angles.
4. Enter the **Circle** command. Specify Point O as the center and specify the radius by selecting the first division point on Line OB. Continue drawing concentric circles in a similar fashion. Locate the center of the second circle at Point O and specify the radius as the distance to the second division point on Line OB. Draw the remaining circles.
5. The points of intersection of the concentric circles and arrayed lines are points on the spiral curve.
6. Enter the **Spline** command and draw a curve connecting the points. Specify the first point as Point O. Specify the second point as the point where the first circle (the smallest circle) intersects the first radial line. Refer to **Figure 7-23**. Continue picking points to define the curve. Specify the default start and end tangents to complete the curve.

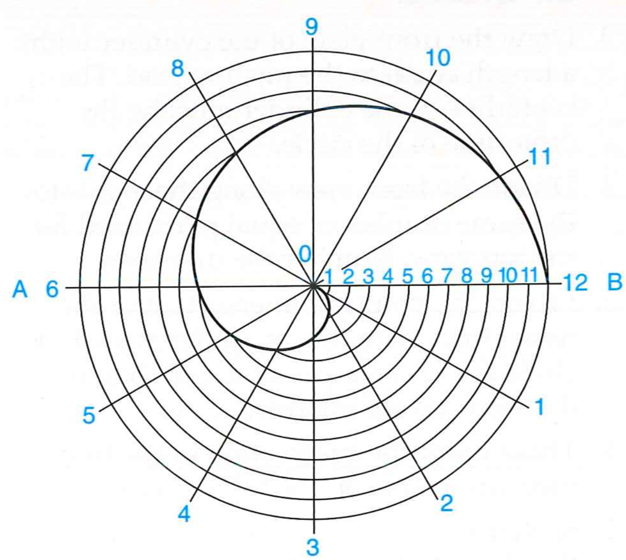



**Figure 7-23.** A spiral of Archimedes curve is formed by a point moving uniformly around and away from a fixed point.

## The Helix

A helix is similar to a spiral, but it is a three-dimensional curve rather than a plane curve. A ***helix*** can be described as a point moving around the circumference of a cylinder at a uniform rate and parallel to the axis of the cylinder. The ***pitch*** or ***lead*** of a helix is the distance, parallel to the cylinder's axis, that it takes for the curve to make one complete revolution on the circumference of the cylinder.

All basic screw threads are based on a helix. Typical uses of the helix are bolt and screw threads, auger bits used in boring wood, flutes on a drill, and helical gears.

## CAD Construct a Helix

### Using Instruments (Manual Procedure)

1. The diameter of the cylinder and the pitch or lead are given, **Figure 7-24A**.
2. Draw the top view as a circle equal to the diameter of the cylinder and divide into any number of equal parts (for example, 12 parts of 30° each). Number the divisions.
3. Draw the front view of the cylinder with a length equal to the pitch or lead. The centerline of the cylinder must be the centerline of the circle.
4. Divide the front view along the axis into the same number of equal parts used for the top view. Number the divisions.
5. Project the points of intersection of the radial lines with the circumference of the circle to the corresponding numbered divisions on the cylinder.
6. These points of intersection in the front view are points on the helix curve.
7. Sketch a light line through the points. Finish with the aid of an irregular curve.

A "stretchout" of the development of a helix is shown in **Figure 7-24B**. Note that the stretchout appears as a right triangle. The helix shown in **Figure 7-24** is a right-hand helix and advances into the work or mating part when turned clockwise. On a left-hand helix, the path moves from right to left and advances into the work or mating part when turned counterclockwise.

### Using the Xline, Line, and Spline Commands (CAD Procedure)

Some CAD programs with 3D drawing capability provide special commands for drawing helix objects as solids. A path curve is first drawn to define the helical shape, and the path is used to "sweep" another object (such as a circle) to create the solid. The path curve establishes the base radius, top radius, turn height (pitch), number of turns, and other parameters of the shape. A cylindrical helix has the same radii for the base and top.

In 2D drawing applications, a helix can be constructed using the same projection methods used in manual drafting. The following procedure is used to project a 2D view of a helix. This procedure uses the **Xline** command to draw construction lines for projecting points between views. ***Construction lines*** are used in drafting to locate points and lay out drawings. By drawing construction lines with the **Xline** command, you can easily differentiate the lines from object lines. You may find it useful to create a separate layer for construction lines and freeze or "hide" the layer when finished to clean up the drawing.

1. The diameter of the cylinder and the pitch or lead are given. Refer to **Figure 7-24A**.
2. Enter the **Circle** command and draw the top view. Specify a center point and enter the **Diameter** option. Enter the diameter of the cylinder.
3. Enter the **Divide** command. Divide the circle into 12 parts.
4. Enter the **Xline** command. Draw lines from the sixth and twelfth division points downward to establish the lines for the cylinder in the front view. Refer to **Figure 7-24A**. Use the Node object

snap to specify the first point of each construction line and use Ortho mode to specify the second point. Construction lines drawn with the **Xline** command are infinite in length and pass through the first and second points you specify. If you use Ortho mode, the line is drawn in a vertical or horizontal direction after specifying the first point.

5. Draw the front view of the cylinder. Enter the **Line** command and draw the top horizontal line of the cylinder through the vertical construction lines. Enter the **Trim** command and trim the line to the edges of the construction lines. Enter the **Offset** command and offset this line to create the baseline of the cylinder. The offset distance is equal to the pitch or lead. To complete the cylinder, enter the **Line** command and use the Endpoint object snap to draw the two vertical lines.
6. Enter the **Divide** command. Divide the vertical line on the left in the front view into the same number of equal parts used for the top view (12). Enter the **Xline** command and draw a horizontal construction line extending from one of the division points. Enter the **Copy** command and copy this line to the other division points.
7. Enter the **Xline** command and draw vertical construction lines to project the division points from the top view to the front view. The points of intersection between the vertical and horizontal lines in the front view are points on the helix curve.
8. Enter the **Spline** command. Pick the points of intersection to define the curve. Specify the default start and end tangents to complete the curve.

## Cycloids

A ***cycloid*** is formed by the path of a fixed point on the circumference of a rolling circle. Cycloids are useful in the design of cycloidal gear teeth. When the circle rolls along a straight line, the path of the fixed point forms a cycloid.

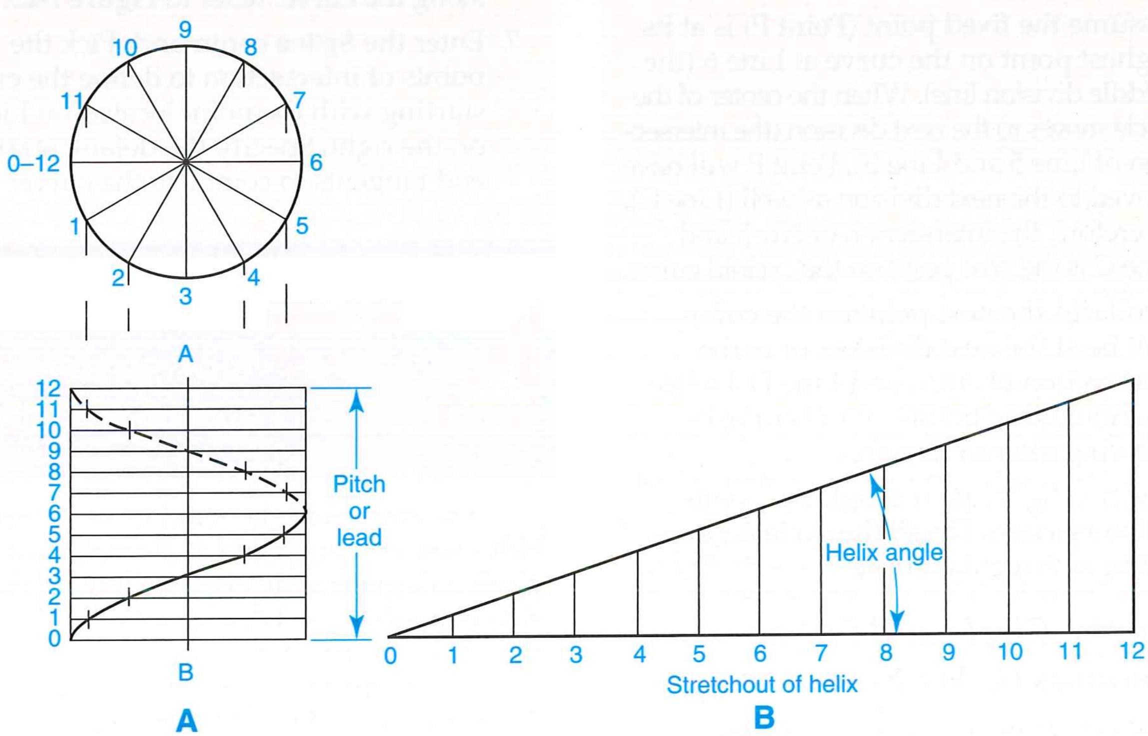



**Figure 7-24**. Constructing a helix. A—The top and front views are drawn, and points are projected between views to establish points on the curve. B—The stretchout of a helix appears as a right triangle.

## Construct a Cycloid

### *Using Instruments (Manual Procedure)*

1. The generating circle and a tangent line, Line AB, are given, **Figure 7-25**. Line AB is equal to the rectified length of the generating circle.
2. Divide the circle and Line AB into the same number of equal parts.
3. Draw Lines C, D, E, F, and G through the division points on the circle and parallel to Line AB.
4. Project the division points on Line AB to Line E (the line that passes through the center of the given circle) by drawing perpendiculars.
5. Using the intersections of the perpendicular lines with Line E as centers and Radius OP, use a compass to draw arcs representing the various positions of the rolling circle as it moves to the left.
6. Assume the fixed point (Point P) is at its highest point on the curve at Line 6 (the middle division line). When the center of the circle moves to the next division (the intersection of Line 5 and Line E), Point P will have moved to the next division as well (Line C). Therefore, the intersection of Arc 5 and Line C is the next point on the cycloid curve.
7. Similarly, the next point on the curve will be at the next division, or at the intersection of Arc 4 and Line D. Locate the remaining points on the curve by drawing intersecting arcs.
8. Sketch a light line through the points of intersection. Finish the cycloid curve using an irregular curve.

### *Using the Circle and Spline Commands (CAD Procedure)*

1. The circle and a tangent line equal to the rectified length (Line AB) are given. Refer to **Figure 7-25**.
2. Enter the **Divide** command. Divide the circle and Line AB into the same number of equal parts.
3. Enter the **Xline** command. Draw Line C through the division points on the circle and parallel to Line AB. Use Ortho mode or the Parallel object snap. Enter the **Copy** command and copy this line to create Lines D, E, F, and G.
4. Enter the **Xline** command. Draw a perpendicular construction line from Point A through Line E. Use Ortho mode or the Perpendicular object snap. Enter the **Copy** command and copy this line to the other division points on Line AB.
5. Enter the **Circle** command. Locate the center at the intersection of Line 1 and Line E. To specify the radius, use the **Distance Between Two Points** calculator function and select Points O and P. Enter the **Copy** command and use the Center object snap to copy this circle to the other intersection points on Line E.
6. The points where the circles intersect with Lines C, D, E, F, and G locate the points along the curve. Refer to **Figure 7-25**.
7. Enter the **Spline** command. Pick the points of intersection to define the curve starting with the point located on Line G on the right. Specify the default start and end tangents to complete the curve.

## Construct an Epicycloid

An epicycloid is similar to a cycloid. However, an ***epicycloid*** is formed by a fixed point on a generating circle rolling on the outside of another circle as opposed to a straight line. The construction is similar to that of the cycloid, except that concentric circle arcs are used instead of Line AB and the other horizontal lines to locate points, **Figure 7-26**.

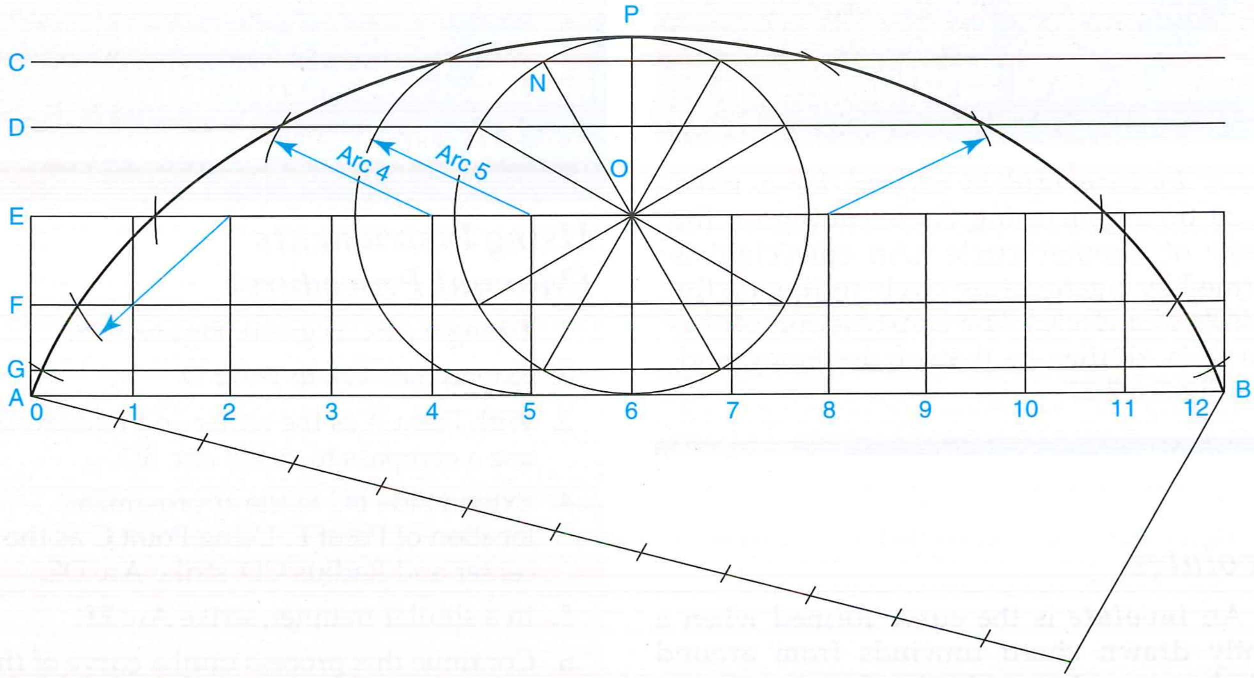



**Figure 7-25**. Constructing a cycloid.

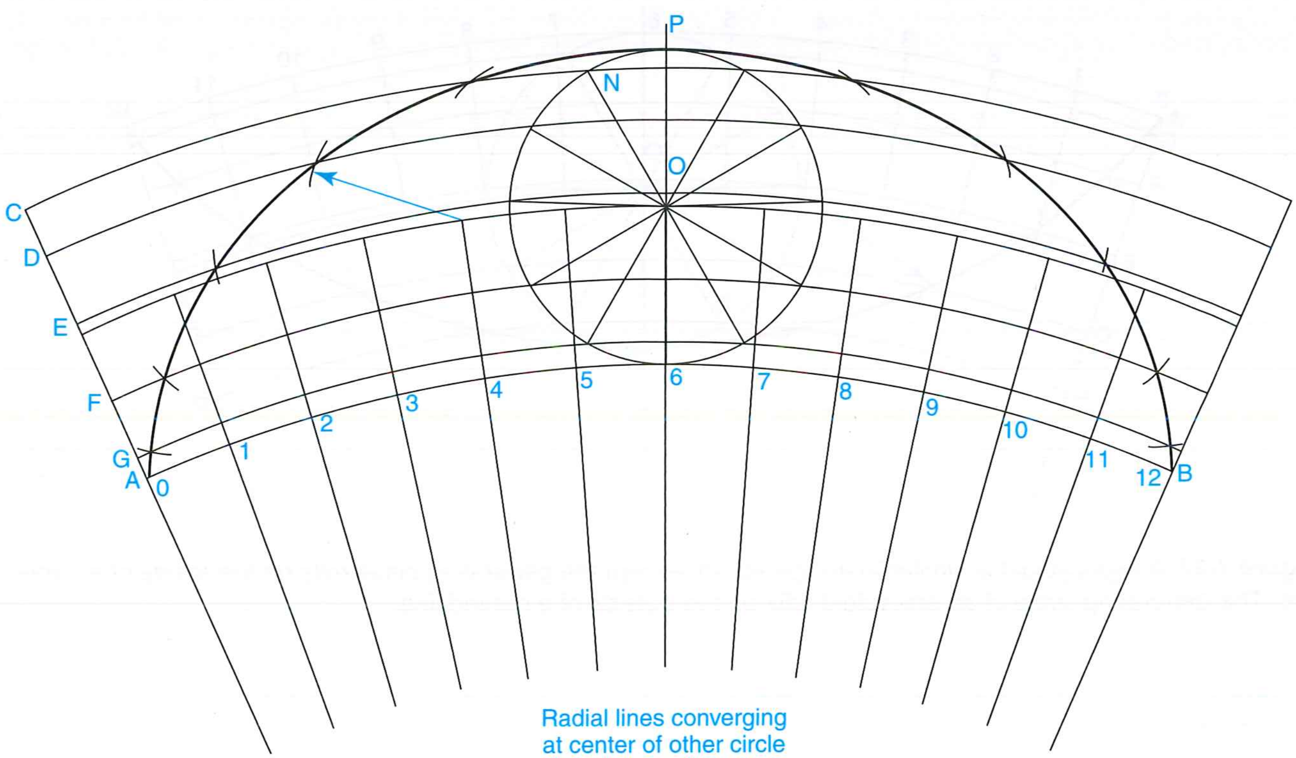



**Figure 7-26**. An epicycloid is similar to a cycloid, except the generating circle rolls on a curved line as opposed to a straight line.

## Construct a Hypocycloid

A ***hypocycloid*** is formed by a fixed point on a generating circle rolling on the *inside* of another circle. (An epicycloid is formed by a generating circle rolling on the *outside* of a circle.) The construction of this curve is similar to that of the epicycloid. See **Figure 7-27**.

## Involutes

An ***involute*** is the curve formed when a tightly drawn chord unwinds from around a circle or a polygon. An involute curve may start on the surface of the circle or polygon, or it may begin a distance away from the geometric form.

## Construct an Involute of an Equilateral Triangle

### *Using Instruments (Manual Procedure)*

1. Triangle ABC is given, **Figure 7-28.**
2. Extend Side CA to Point D.
3. With Point A as the center and Radius AB, use a compass to strike Arc BD.
4. Extend Side BC to the approximate location of Point E. Using Point C as the center and Radius CD, strike Arc DE.
5. In a similar manner, strike Arc EF.
6. Continue this process until a curve of the desired size is completed.

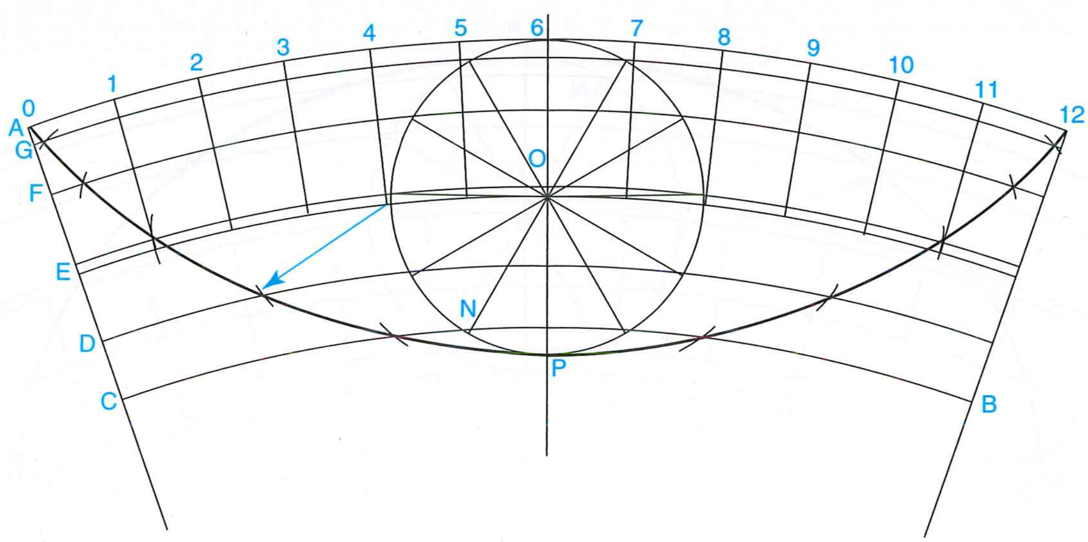



**Figure 7-27.** A hypocycloid is similar to an epicycloid, except the generating circle rolls on the *inside* of a curved line. The generating circle of an epicycloid rolls on the *outside* of a curved line.

## *Using the Arc and Extend Commands (CAD Procedure)*

The **Arc** command can be used to construct an involute of an equilateral triangle by using the **Start, Center, Angle** option. This option allows you to specify the start point, center point, and included angle of the arc. The ***included angle*** is the angle formed by two lines connecting the endpoints of the arc to the center point. With the **Start, Center, Angle** option, specifying a positive angle draws the arc counterclockwise. Specifying a negative angle draws the arc clockwise. The following procedure is used to construct an involute of an equilateral triangle by drawing arcs and extending lines with the **Extend** command.

1. Triangle ABC is given. Refer to **Figure 7-28**.
2. Enter the **Copy** command. Copy Side CA to create Line AD.
3. Enter the **Arc** command. To draw Arc BD, enter the **Start, Center, Angle** option. Specify Point B as the start point and Point A as the center point. Specify the angle as –120°. This angle will draw the arc in a clockwise direction and is calculated as the supplementary angle of Angle CAB (an equilateral triangle has three equal 60° angles and 180° – 60° = 120°). Refer to **Figure 7-28**.
4. Enter the **Arc** command. To draw Arc DE, enter the **Start, Center, Angle** option. Specify Point D as the start point and Point C as the center point. Specify the angle as –120°. This is calculated as the supplementary angle of Angle ACB.
5. Enter the **Extend** command. Select Arc DE as the boundary edge. Extend Line BC to create Line CE.
6. In a similar manner, create Arc EF and extend Line AB to create Line BF. Continue this process until a curve of the desired size is completed.

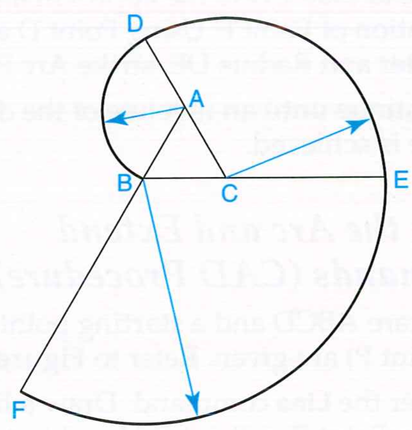



**Figure 7-28.** Constructing an involute of an equilateral triangle.

## Construct an Involute of a Square

### *Using Instruments (Manual Procedure)*

1. Square ABCD and a starting point (Point P) are given, **Figure 7-29**.
2. Extend Side AB to Point P and Side DA to the approximate location of Point E. Using Point A as the center and Radius AP, strike Arc PE.
3. Extend Side CD to the approximate location of Point F. Using Point D as the center and Radius DE, strike Arc EF.
4. Continue until an involute of the desired size is achieved.

### *Using the Arc and Extend Commands (CAD Procedure)*

1. Square ABCD and a starting point (Point P) are given. Refer to **Figure 7-29**.
2. Enter the **Line** command. Draw a line from Point B to Point P. Use object snaps.
3. Enter the **Arc** command. To draw Arc PE, enter the **Start, Center, Angle** option. Specify Point P as the start point and Point A as the center point. Specify the angle as –90°. This angle will draw the arc in a clockwise direction and is calculated as a right angle (Angle PAE is formed by two perpendicular lines). Refer to **Figure 7-29**.
4. Enter the **Extend** command. Select Arc PE as the boundary edge. Extend Line DA to create Line AE.
5. Enter the **Arc** command. To draw Arc EF, enter the **Start, Center, Angle** option. Specify Point E as the start point and Point D as the center point. Specify the angle as –90°. This angle is calculated as a right angle (Angle EDF is formed by two perpendicular lines).
6. Enter the **Extend** command. Select Arc EF as the boundary edge. Extend Line CD to create Line DF.
7. In a similar manner, create Arc FG and extend Line BC to create Line CG. Continue this process until a curve of the desired size is completed.

## Construct an Involute of a Circle

### *Using Instruments (Manual Procedure)*

1. Circle O and the starting point (Point P) are given, **Figure 7-30**.
2. Divide the circle into a number of equal parts. Draw tangents at the division points.
3. Beginning at Point A, the first division point on the circle clockwise from Point P, lay off on Tangent A a distance equal to the length of Arc AP.

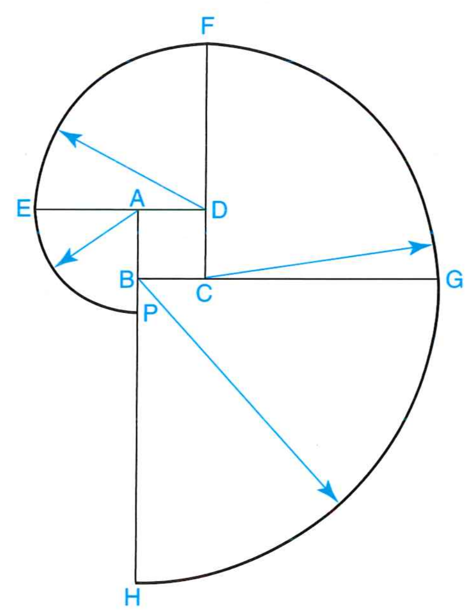



**Figure 7-29.** Constructing an involute of a square.

4. On Tangent B, lay off a distance equal to the length of Arc AP + Arc BA (the length of two circle arcs).
5. Continue with Tangent C with a distance equal to three circle arcs, and so on until the distance on the final tangent has been set off.
6. Sketch a light line through these points. Finish the involute of the circle using an irregular curve.

The involute of a circle is the curve form used in the design of involute gear teeth, **Figure 7-31**.

## *Using the Offset and Arc Commands (CAD Procedure)*

1. Circle O and the starting point (Point P) are given. Refer to **Figure 7-30**.
2. Enter the **Line** command. Draw a line from the center point of the circle to Point P. Use object snaps.
3. Enter the **Array** command. Select the line and create a polar array. Specify the number of objects as 12 and specify the center point as the center point of the circle. Specify an angular rotation of 360° and rotate the objects as they are copied. This creates a pattern of 30° lines and divides the circle into 12 parts.
4. Use the **Offset** command to offset the radial lines. Offset each line at a distance equal to the length of the corresponding circle arc. Refer to **Figure 7-30**. For example, Line A should be offset at a distance equal to the length of Arc AP and Line B should be offset at a distance equal to the length of Arc BP. To determine the length of each arc when entering the offset distance, use the system calculator. Multiply the circumference of the circle by the fractional portion of the circle arc on the circumference. For example, the length of Arc AP is equal to the circumference multiplied by 1/12 (30° ÷ 360° = 1/12). The circumference of the circle can be entered into the system calculator directly by using the **Properties** command and selecting the circle.
5. The endpoints of the offset lines locate the points on the involute curve.
6. Enter the **Arc** command to draw arcs to construct the curve. Use the default **3 Points** option and draw a series of three-point arcs clockwise. Draw the first arc by selecting Point P as the start point. Select the endpoint of the offset of Line A as the second point, and select the endpoint of the offset of Line B as the third point. Continue drawing arcs to finish the involute curve.

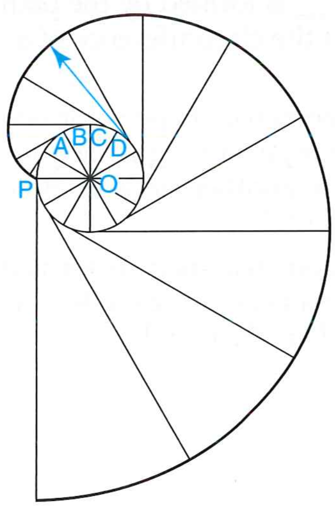



**Figure 7-30**. Constructing an involute of a circle.

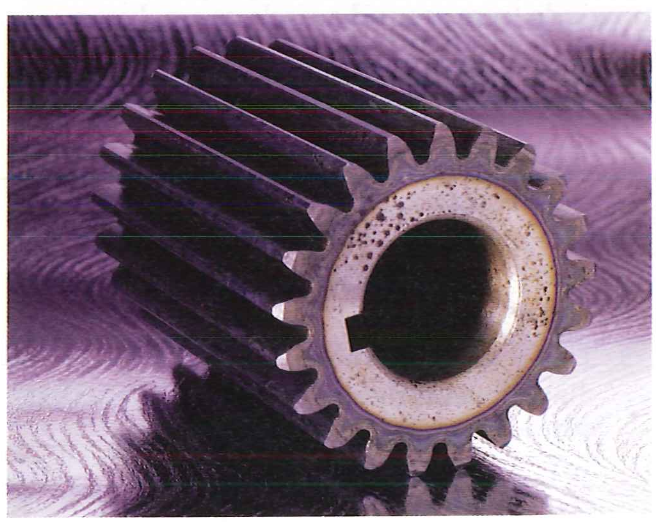


**Figure 7-31**. Gears are commonly designed with involute curve teeth.

## Chapter Summary

Advanced geometric constructions are difficult to solve as drafting problems. Examples of advanced constructions include conic sections, the spiral of Archimedes, the helix, cycloids, and involutes. These constructions can be drawn using manual or CAD methods.

Conic sections are curved shapes produced by passing a plane through a right circular cone. A right circular cone has a circular base and an axis perpendicular to the base at its center. Four types of curves result from cutting planes at different angles. These are the circle, ellipse, parabola, and hyperbola. A circle is formed when a plane is passed through the cone perpendicular to the vertical axis. An ellipse is formed when a plane is passed through the cone at an angle greater than that of the elements. A parabola is formed when a plane cuts a right circular cone at the same angle to the axis as the elements. A hyperbola is formed when a plane cuts two right circular cones joined at their base.

In addition to conic sections, other advanced curves are commonly drawn by drafters. A spiral of Archimedes is formed by a point moving uniformly around and away from a fixed point. A helix is a three-dimensional curve in the shape of a spiral. It can be described as a point moving around the circumference of a cylinder at a uniform rate and parallel to the cylinder's axis. A cycloid is formed by the path of a fixed point on the circumference of a rolling circle. An involute is a curve formed when a tightly drawn chord unwinds from around a circle or a polygon.

## Review Questions

1. Conic sections are curved shapes produced by passing a cutting plane through a right circular _____.
2. A(n) _____ results when a circle is viewed at an angle.
3. The _____ axis of an ellipse is the largest diameter and the _____ axis is the smallest diameter.
4. What is a *trammel*?
5. What CAD command greatly simplifies the task of drawing ellipses?
6. What option of the CAD command identified in Question 5 allows you to draw elliptical arcs by specifying the start and end angles?
7. In manual drafting, considerable time can be saved in ellipse construction by using a(n) _____ template.
8. What geometric shape is formed when a plane cuts a right circular cone at the same angle as the elements?
9. What CAD command is used to draw curves called *splines*?
10. The _____ of a hyperbola are lines that intersect at the midpoint of the transverse axis.
11. Using CAD, you can draw a hyperbola using the _____ command to draw a curve through a series of plotted points.
12. When the asymptotes are at right angles to each other, the resulting hyperbola is called a(n) _____ hyperbola or a rectangular hyperbola.
13. The spiral of Archimedes curve is used in the design of cams to change uniform _____ motion into uniform _____ motion.
14. What geometric shape is similar to a spiral, but is a three-dimensional curve rather than a plane curve?
15. A(n) _____ is formed by the path of a fixed point on the circumference of a rolling circle.
16. What geometric shape is formed by a fixed point on a generating circle rolling on the outside of another circle as opposed to a straight line?
17. What geometric shape is formed by a fixed point on a generating circle rolling on the inside of another circle?
18. What is an *involute*?

## *Problems and Activities*

The following problems involve complex geometric constructions. Practical applications are included to acquaint you with typical geometric problems the drafter, designer, or engineer must solve.

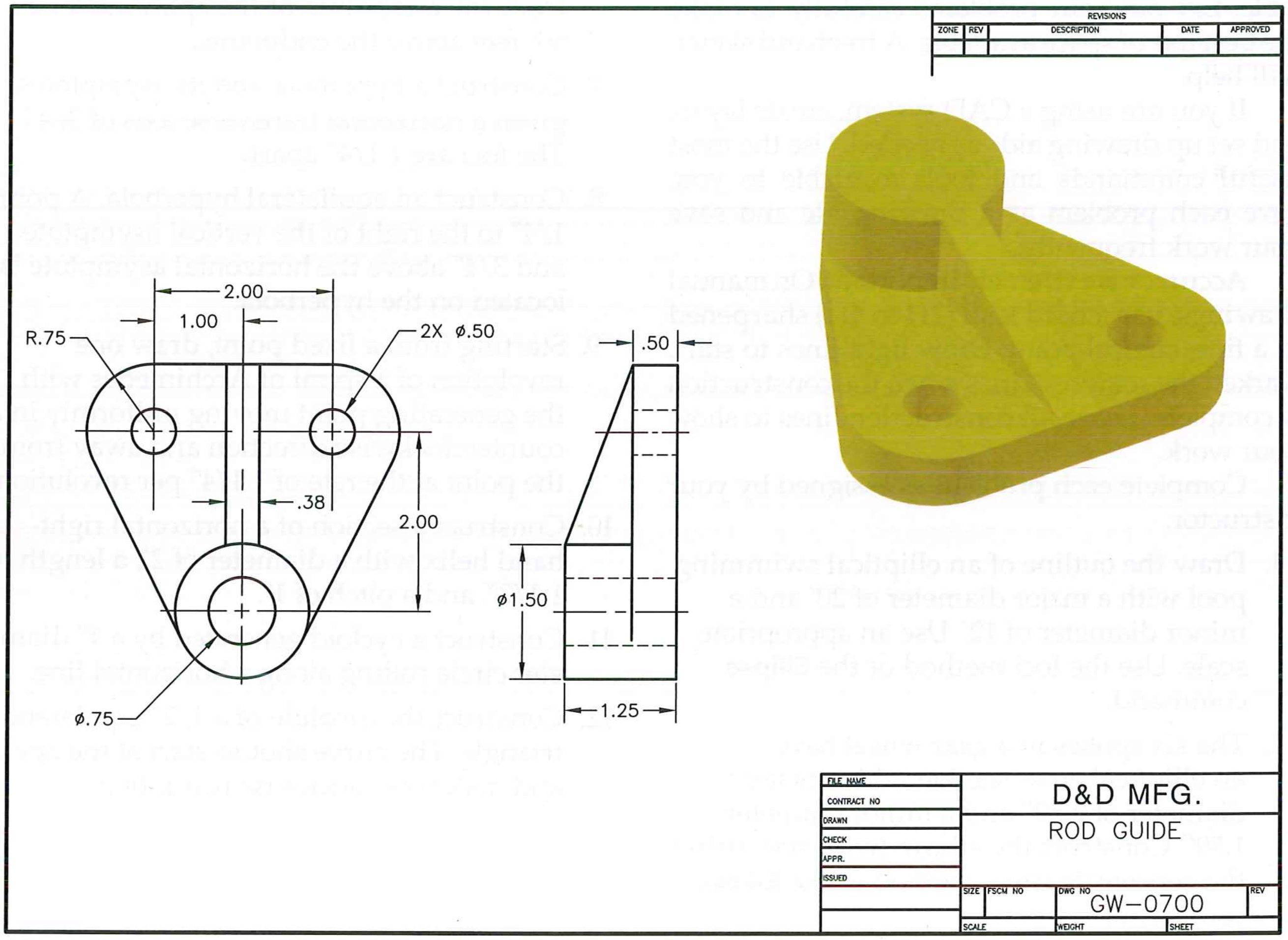
If you are drawing problems manually, use a suitable sheet size and the Layout I sheet format given in the Reference Section. Place drawing sheets horizontally on the drawing board or table. Lay out your problems carefully to make the best use of space available. A freehand sketch will help.

If you are using a CAD system, create layers and set up drawing aids as needed. Use the most useful commands and tools available to you. Save each problem as a drawing file and save your work frequently.

Accuracy is extremely important. On manual drawings, use a hard lead (2H to 4H) sharpened to a fine, conical point. Draw light lines to start. Darken the required lines when the construction is complete. Leave all construction lines to show your work.

Complete each problem as assigned by your instructor.

1. Draw the outline of an elliptical swimming pool with a major diameter of 20′ and a minor diameter of 12′. Use an appropriate scale. Use the foci method or the **Ellipse** command.
2. The six spokes in a gear wheel have an elliptical cross section with a major diameter of 2.50″ and a minor diameter of 1.50″. Construct the ellipse twice size, using the concentric circle method or the **Ellipse** command.
3. The design for a bridge support arch is elliptical in shape. It has a span of 36′ (the major diameter), and the rise at the center of the ellipse is 12′ above the major diameter. Construct the half ellipse representing the arch. Use the trammel method or the **Ellipse** command.
4. Using an ellipse template, draw an ellipse. Label the size and angle of the ellipse.
5. Construct a parabola using the focus method. Use a compass and irregular curve or the **Spline** command. The distance from the directrix to the focus is 1 1/4″.
6. A highway overpass has a horizontal span of 200′ and a rise of 25′. The curve form is parabolic. Draw the form of this curve. *Hint:* The apex of the two tangent lines from the endpoints of the span must be 50′ feet above the endpoints.
7. Construct a hyperbola and its asymptotes, given a horizontal transverse axis of 3/4″. The foci are 1 1/4″ apart.
8. Construct an equilateral hyperbola. A point 1/4″ to the right of the vertical asymptote and 3/4″ above the horizontal asymptote is located on the hyperbola.
9. Starting from a fixed point, draw one revolution of a spiral of Archimedes with the generating point moving uniformly in a counterclockwise direction and away from the point at the rate of 1 1/4″ per revolution.
10. Construct a section of a horizontal right-hand helix with a diameter of 2″, a length of 1 1/2″, and a pitch of 1″.
11. Construct a cycloid generated by a 1″ diameter circle rolling along a horizontal line.
12. Construct the involute of a 1/2″ equilateral triangle. The curve should start at the apex and make one clockwise revolution.




Multiview drawings are widely used in drafting. A multiview drawing includes several views to describe the features of a three-dimensional object in two dimensions.